

Mathematical studies SL

Graphic display calculators

First examinations 2006

Diploma Programme

Teacher support material



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International Baccalaureate Organization

Buenos Aires

Cardiff

Geneva

New York

Singapore

Diploma Programme
Mathematical studies SL (graphic display
calculators)—teacher support material

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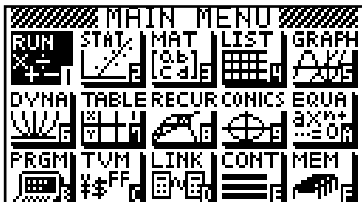
Introduction

This document gives guidance to teachers and students using graphic display calculators (GDCs). Many different types of GDC are available, but this document provides guidance for two models—the Casio 9850 GB+ and the Texas Instruments TI-83+.

The guidance focuses on the main features of each model, using specimen questions and examples to highlight these main features.

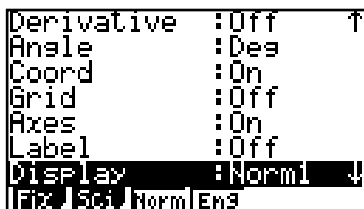
Main features

Run mode



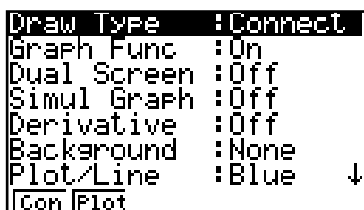
Press "Set up" on the GDC.

The GDC must always be in "Degree" mode.



- "Derivative" needs to be "On" for analysis of functions.
- "Fix" gives the number of figures that occur after the decimal point.
- "Sci" means "scientific", and is useful for standard form. Students must be able to transfer this notation back to mathematical notation.

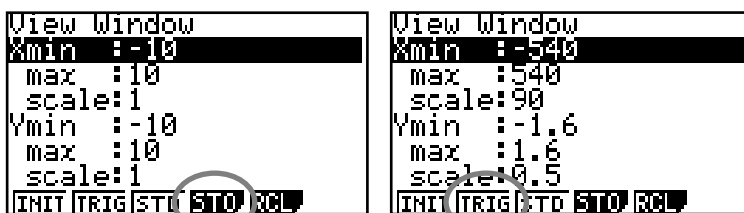
Graph mode/set up



- "Draw Type" draws a graph either as a continuous function or as dots.
- It draws graphs one by one or simultaneously.
- "Dual Screen" needs to be "On" to draw both graph and table.
- When "Dual Screen" is on "G-T" splits the screen vertically. The left side is the graph and the right side is the table of values.

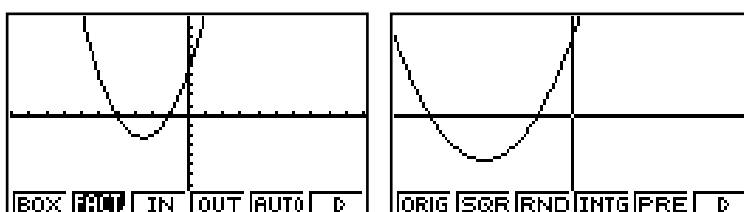
"F6" draws the graph of "y = ".

- In "View Window", the user enters the required domain and range (x- and y-values).



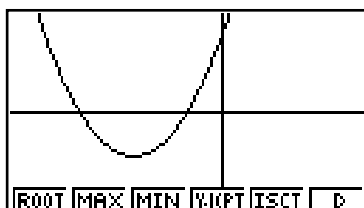
- "Standard (STD)" is a good starting point for most functions.
- "TRIG" is to be used for periodic functions.

Press "Zoom" on the GDC.



Press "Trace" on the GDC to move the cursor along the graph.

Press "G-SOLV" to calculate numerical values.



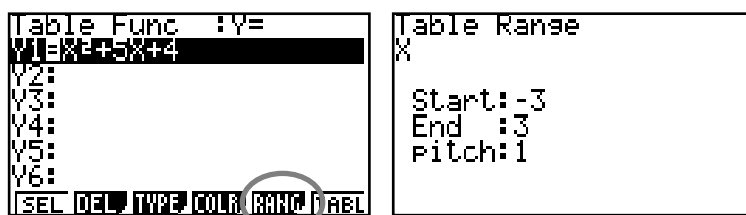
Using "G-SOLV"

- For the x-intercept, place the cursor to the left of the intersection point, press "EXE", then place the cursor to the right of the intersection point, press "EXE". Press "EXE" once more to display the zero value.
- "MAX" is used to calculate the maximum turning point (appears as above for zeros); "MIN" is used to calculate the minimum turning point (appears as above for zeros).
- For the y-intercept, "ISCT" is used to calculate where two curves intersect.

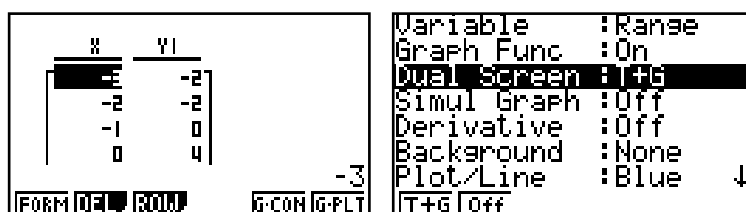
"X-calc" and "Y-calc" calculate individual values.

Table mode

"F6" displays the table of values for the equation entered in "y =".

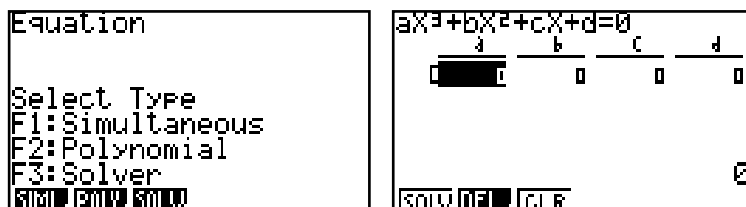


- Use "Range" to enter the required domain (x -values).
- Enter the x -values directly on to the screen to find the corresponding y -values.



- Change "Dual Screen" to "T+G" for table and graph.

Equation mode



- Use "Solver" for algebraic equations.
- Use "Polynomial" to input coefficients for quadratic and cubic equations.
- Use "Simultaneous" to input coefficients for linear simultaneous equations.

Recur mode

Use "Recur" for terms of sequences and sums of series.

The Recur mode interface consists of several screens:

- Settings Screen:** Shows options like `Display : On`, `Draw Type : Connect`, `Graph Func : On`, `Dual Screen : Off`, `Simul Graph : Off`, `Background : None`, `Plot/Line : Blue`, and `On/Off`.
- Select Type Screen:** Lists three recursion types: `F1: an=An+B`, `F2: an+1=Aan+Bn+C`, and `F3: an+2=Aan+1+Ban+...`. The first option is circled.
- Recursion Definition Screen:** Shows `an+1=7n+8` and `an+1:` with a cursor. The `TYPE` and `TABLE` options at the bottom are circled.
- Table Range Screen:** Shows `Table Range n`, `Start: 1`, and `End: 20`.
- Table Output Screen:** Displays a table with columns `n+1`, `an+1`, and `Σan+1`. The values are:

n+1	an+1	Σan+1
17	120	1088
18	127	1215
19	134	1349
20	141	1490

Matrix mode

Use "Matrix" for chi-squared analysis.

- Enter contingency table values for chi-squared calculation, then use "STAT-TEST".

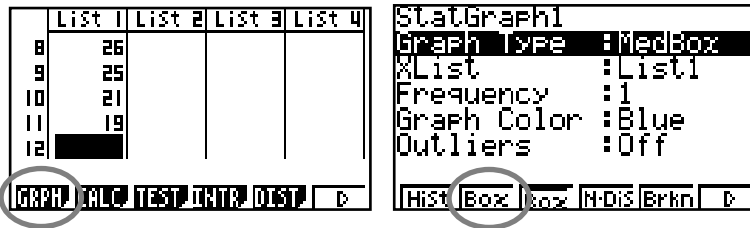
The Matrix mode interface includes the following screens:

- Matrix Selection Screen:** Lists `Mat A : 3x 2`, `Mat B : None`, `Mat C : None`, `Mat D : None`, `Mat E : None`, and `Mat F : None`.
- Matrix Data Entry Screen:** Shows a 3x2 matrix `A` with values:

1	24	36
2	51	12
3	19	16
- Chi-Squared Test Screen:** Shows `χ² Test`, `Observed: Mat A`, and `Execute`. Below are columns labeled `A`, `B`, `C`, `D`, `E`, and `D`.
- Matrix Results Screen:** Lists `Mat U : None`, `Mat W : None`, `Mat X : None`, `Mat Y : None`, `Mat Z : None`, and `Mat Ans : 3x 2`.

The expected values appear at the bottom of the list ("Mat Ans") after the chi-squared test has been performed in "STAT" mode.

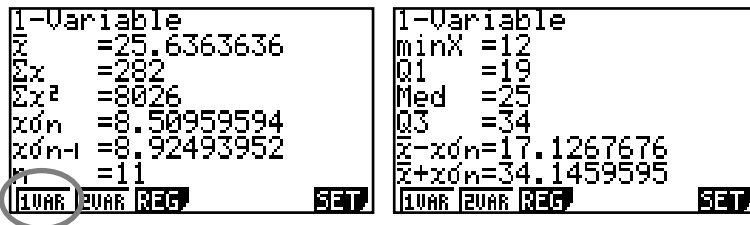
Statistics mode



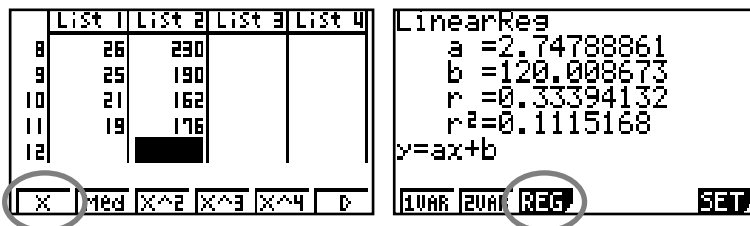
- Use “GRPH” to set the type of graph and the data source.
- Use “HiSt” and “Box” for histograms and box plots.

One-variable summary statistics

- Use “CALC” for one-variable summary statistics.



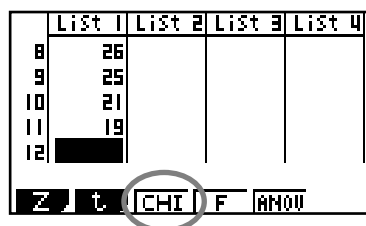
Two-variable statistics



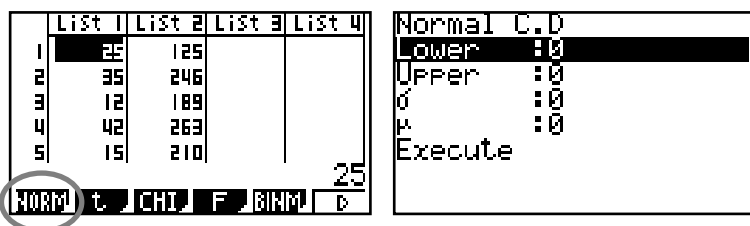
Least squares regression

Regression and correlation

- For the chi-squared test, enter data as a matrix.



- Use “DISTNORM” to calculate normal distributions.



TVM

Press "F1" for simple interest.

Press "F2" for compound interest.

<pre>Simple Interest:365 n = 120 I% = 5 PV = -8000 SI SFV</pre>	<pre>Compound Interest:End n = 120 I% = 5 PV = -8000 PMT=0 FV = 0 P/Y=12 n I% PV PMT FV AMT</pre>
---	---

- Enter "PV" as negative.
- "n" = the total number of payment periods.
- "I%" = the annual interest rate.
- "PV" = the present value, expressed as a negative to represent cash outflow (investment).
- "PMT" = the payment amount.
- "FV" = the future value.
- "P/Y" = the number of payment periods per year.
- "C/Y" = the number of compound periods per year.

Compound interest problems

Because there are no payments when solving compound interest problems, "PMT" must be set to "0" and "C/Y" must be set to the correct number of compound periods per year (see the financial mathematics examples on pages 17–18).

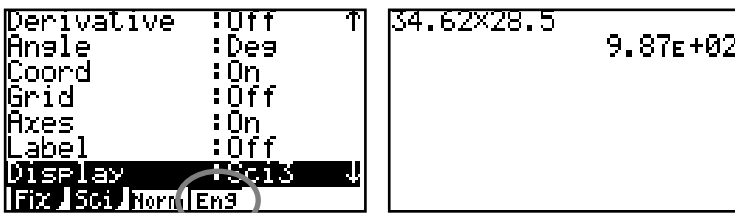
Examples of the main features in use for the mathematical studies SL course

Scientific mode

Found in run mode; set up the screen.

Question

Calculate 34.62×28.5 and write your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.



Students must realize that E+02 means 10^2 and write their answer using the correct mathematical terminology.

If the question asks for the answer to be given correct to two decimal places, the student's answer should be:

$$(34.62 \times 28.5) = 9.87 \times 10^2.$$

Arithmetic progression/geometric progression

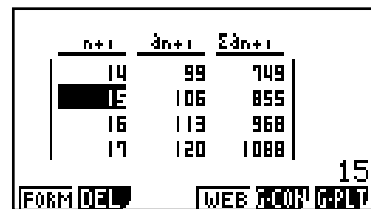
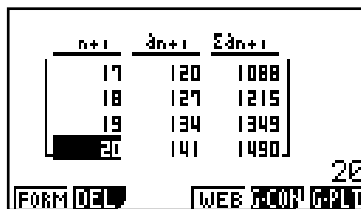
Question

The first term of an arithmetic progression is 8, the common difference is 7.

- (a) Find the 20th term.
- (b) Find the sum of 15 terms.



Recursion



(a)

Student's answer:

$$U_{20} = 8 + (19 \times 7) = 141.$$

(b)

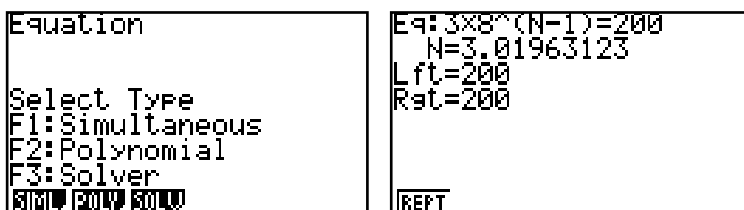
Student's answer:

$$S_{15} = 15/2 (2 \times 8 + 14 \times 7) = 855.$$

If students are given a number and are asked to find which term it is, they can use "Solver".

Find which term of the sequence with 3 as first term and common ratio of 8 first exceeds 200.

- Use "Equation", then "Solver".
- Fill in A and B for $AR^{(n-1)} = 200$, then press "EXE".



Student's answer:

$$3 \times 8^{(n-1)} > 200$$

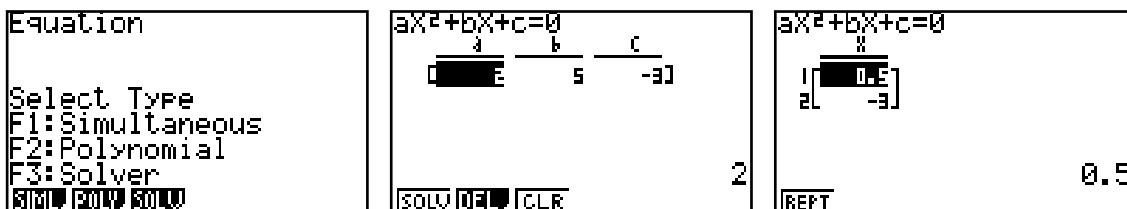
$$n = 4.$$

Solution of quadratics

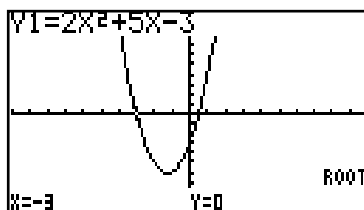
- Use "Equation", then "Polynomial".
- "Degree 2".

Question

Solve for x the equation $2x^2 + 5x - 3 = 0$.



- Alternatively, use "Graph": $Y = 2x^2 + 5x - 3$.
- Check "View Window".
- "Draw".
- "G-SOLV", then "ROOT".



Use "Trace" to find the other root

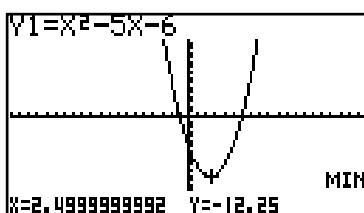
The student may write down the solutions without showing any further working. If any more is required, the question could first ask the student to factorize the function.

Vertex of quadratic

Question

Find the vertex of the function $y = x^2 - 5x - 6$

- Use "G-SOLV", then "MIN".



The student's answer could illustrate at least two possible scenarios depending on what the question-setter is aiming to test.

To show that the vertex is on the axis of symmetry, the student could write:

$$x = (-b/2a) = 2.5 \text{ (5/2)}; \quad y = -12.25.$$

If differentiation is required, the student could write:

$$dy/dx = 2x - 5 = 0 \text{ at stationary values}$$

$$\Rightarrow x = 5/2; y = -12.25.$$

If no method is specified, then both methods are acceptable.

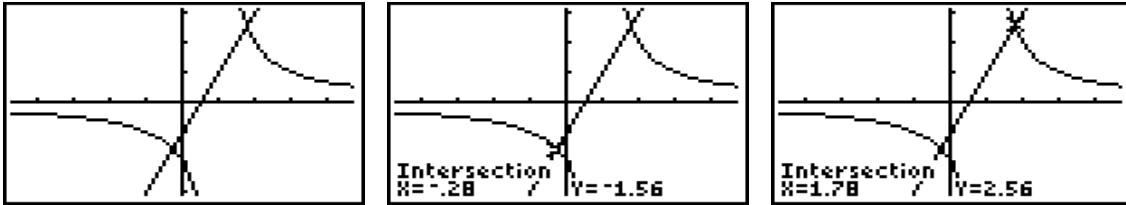
Intersection of curves

- Graph the curves and use the intersection function as above.

Question

Find the points of intersection of the graphs of the following functions:

$$y = 2/(x - 1) \text{ and } y = 2x - 1.$$



It is sufficient for the student to write:

$$2/(x - 1) = 2x - 1 \Rightarrow x = -0.281 \text{ and } 1.78.$$

Exponential equations

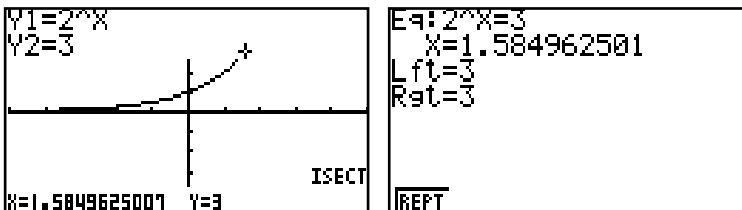
“Graph”: $Y_1 = a^x$.

“Graph”: $Y_2 = b$.

Then use the intersection function as above, or use “Solver”.

Question

Find the value of x for the equation above if $a = 2$ and $b = 3$.



Student’s answer:

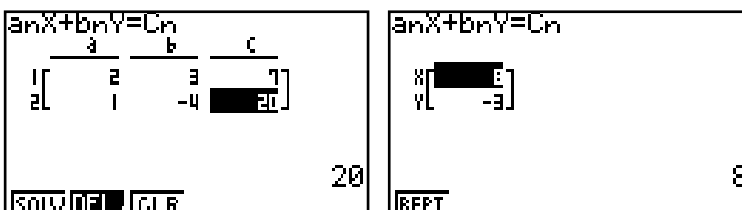
$$2^x = 3 \Rightarrow x = 1.58 \text{ (no working need be shown).}$$

Simultaneous equations

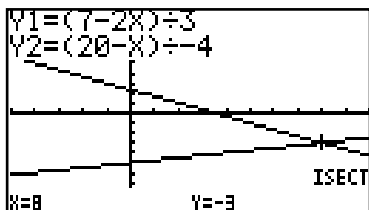
Use “Equation”, then “Simultaneous”.

Question

Solve the equations $2x + 3y = 7$ and $x - 4y = 20$.



- Alternatively, the user can rearrange the equations to $y = (7 - 2x)/3$ and $y = (20 - x)/-4$, then graph and find the intersection point.



Students should write down both equations and give the solutions for x and y .

Student's answer:

$$x = 8, y = -3.$$

Truth tables

- Using "1" for "True" and "0" for "False", enter values in lists for p and q .

Question

Complete the truth table for $p \vee \neg q$.

- Set the cursor on "List" heading.
- Select "Option".
- Press "F1" for list, then "F6" for logic.

	List 3	List 4	List 5	List 6
1	1	1	0	
2	1	0	1	
3	0	1	0	
4	0	0	1	
5				

Not List 4
List L→M Dim Fill Seq

	List 3	List 4	List 5	List 6
1	1	1	0	1
2	1	0	1	1
3	0	1	0	0
4	0	0	1	1
5				

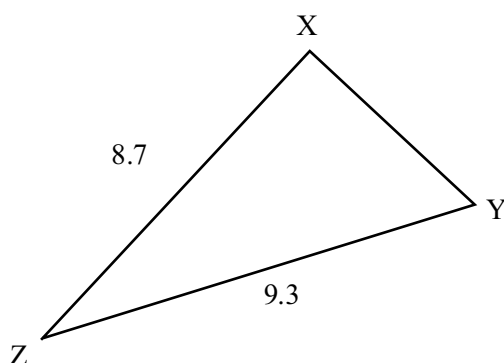
List 3 Or List 5
List L→M Dim Fill Seq

Students should show all the intermediate steps even though the final answer can be obtained from the GDC.

Cosine rule using "Solver"

Question

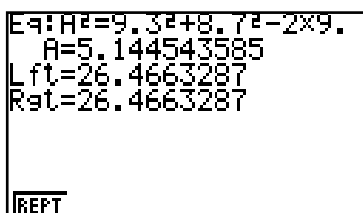
In triangle XYZ, $XZ = 8.7$ cm, $YZ = 9.3$ cm and angle $XZY = 33^\circ$. Find the length of side XY.



Use "Equation", then "Solver".

"ALPHA": "A =".

Alternatively, use "X =".



The student should put the corresponding values into the cosine rule and then write the answer, for example:

$$Z^2 = 9.3^2 + 8.7^2 - 2 \times 9.3 \times 8.7 \cos(33^\circ)$$

$$Z = 5.14.$$

Matrices

These will only be required for entering observed data in the chi-squared distribution. See Matrix mode on page 5 for set-up instructions.

Question

Enter a 2×3 contingency table and perform a chi-squared test.

- Use "Matrix", then "STAT-TEST" for matrices.

A	1	2	3
1	15	26	35
2	24	35	67

67

R-OP ROW COL

χ^2 Test
 $\chi^2=1.1374$
 $P=0.56624$
 $df=2$
 Expected=Mat Ans

Ans	1	2	3
1	22.95	38.376	
2	24.326	38.049	63.623

14.67326733

R-OP ROW COL

Expected values

Unless another type of question is set, this value for chi-squared, the degrees of freedom and the expected values can all be obtained from the GDC. However, students should be familiar with the formula and know how to use it.

Statistics

This option allows the user to enter data in lists, sort the lists in ascending or descending order, and use formulae to generate new lists.

Use "Option".

List 1	List 2	List 3	List 4
1	5	14	14
2	10	21	35
3	14	32	67
4	18	16	83
5	22	8	91

6

GRPH CALC TEST DATA DIST

List 1	List 2	List 3	List 4
1	6	14	14
2	10	21	35
3	14	32	67
4	18	16	83
5	22	8	91

Cuml List 2
 Sum Prod Cuml %

- Use "GRPH" to select type.
- Use "CALC" to calculate various statistics.
- Use "TEST" for chi-squared, t -tests.

Question

Use "CALC" to find the mean, median, standard deviation, etc of a list of numbers:

12, 15, 19, 21, 25, 25, 26, 28, 34, 35.

List 1	List 2	List 3	List 4
1	12	189	
2	15	210	
3	19	176	
4	21	162	
5	25	125	

VAR VAR REG SET

1-Variable
 \bar{x} = 24
 Σx = 240
 Σx^2 = 6262
 $\sigma_{x/n}$ = 7.08519583
 $\sigma_{x/n-1}$ = 7.46845216
 n = 10

VAR VAR REG SET

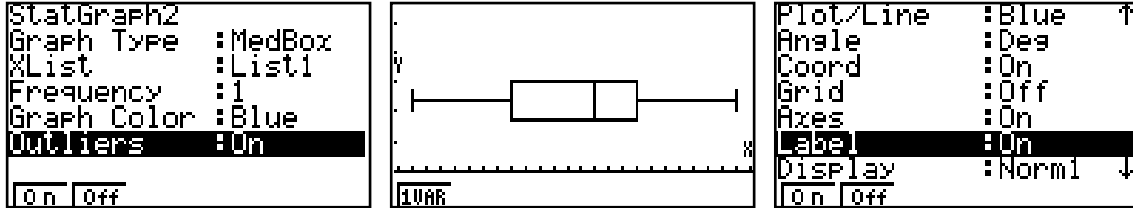
1-Variable
 Q_1 = 19
 Med = 25
 Q_3 = 28
 $\bar{x}-\sigma_{x/n}$ = 16.9148041
 $\bar{x}+\sigma_{x/n}$ = 31.0851958
 $\max X$ = 35

VAR VAR REG SET

For grouped data, using the mid-point of the groups gives an estimate of the mean and standard deviation, but this does **not** give accurate values for the quartiles.

Students may write down the values of the mean and standard deviation from the GDC without showing any further working, but they must be careful to enter the data correctly. A wrong answer from the GDC with no method shown is awarded no marks.

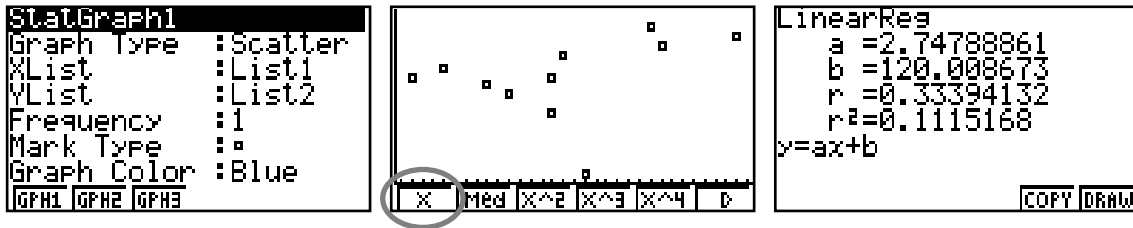
- Use "GRAPH" to view the shape of the data.



- Use "Trace" for a 5-figure summary.
- Use "REG", then select "2" for variable analysis.
- Use "Correlation", then "Regression".

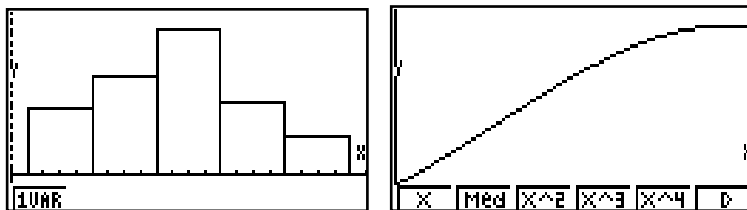
Student's answer:

12, 15, 19, 21, 25, 25, 26, 28, 34, 35
 189, 210, 176, 162, 145, 190, 230, 197, 285, 246.



If data are provided, then the students should be able to write down the correlation coefficient, r , and the equation of the regression line directly from the GDC. However, they should also be familiar with the formulae and how to use them.

- "Frequency Table": "List 1" represents numbers, "List 2" the frequency.
- "Cumulative Frequency": "List 3" is the cumulative sum ("Cuml"). Use "List 2", then "Option".



- Use upper boundaries of class interval.

Calculus

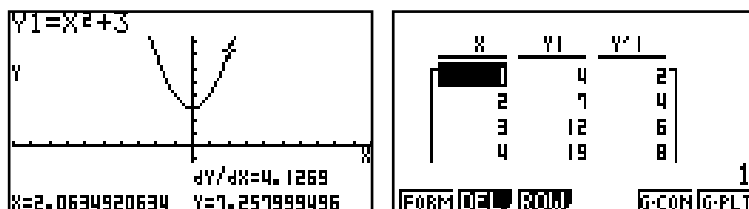
Press "Set up" on the GDC.

Variable	: Range
Graph Func	: On
Dual Screen	: Off
Simul Graph	: Off
Derivative	: On
Background	: None
Plot/Line	: Blue ↓
	: On/Off

- "Derivative" needs to be "On".

To calculate the value of y and dy/dx at any point on a curve

- To draw the curve $y = x^2 + 3$, use "Trace" in graph mode or enter the x -values directly into the table.



Student's answer:

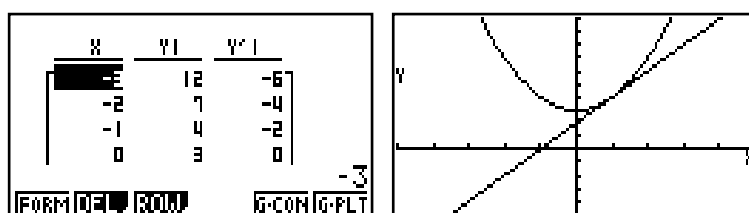
$$dy/dx = 2x$$

$$\text{when } x = 2, dy/dx = 4.$$

Equation of tangent line

Question

Find the equation of the tangent to the curve $y = x^2 + 3$ at the point (1, 4).



Student's answer:

$$dy/dx = 2x$$

$$\text{at } x = 1, dy/dx = 2 (=m)$$

$$y = 2x + c$$

$$4 = 2 \times 1 + c$$

$$c = 2$$

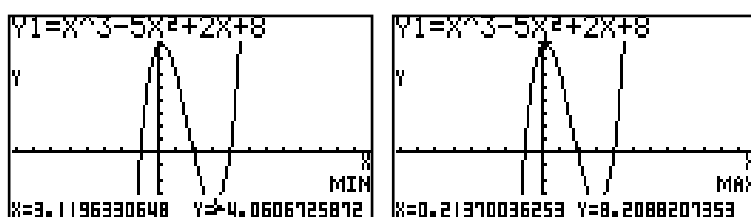
The equation is $y = 2x + 2$.

Finding local maximum and minimum points

Question

Find the local maximum and minimum points on the graph of the curve $y = x^3 - 5x^2 + 2x + 8$.

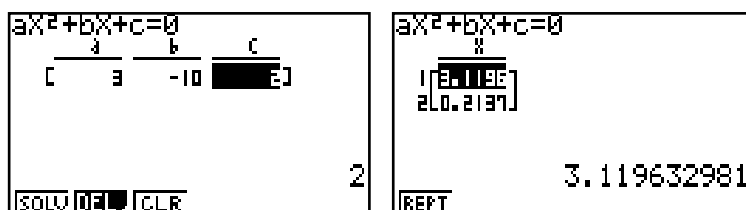
- Enter the equation in "Y1 =", then draw the graph.
- Use "G-SOLV", then "MIN".
- Repeat the process for the maximum point.



Student's answer:

$dy/dx = 3x^2 - 10x + 2 = 0$ at stationary values.

- The student will then have to solve the quadratic using a programme, the solver function, by graphing or by using the quadratic formula.



Student's answer:

$x = 3.12$ or 0.214 .

If students enter the derived equation into Y2, they can then use "CALC" or "Table" to find the y-values.

Financial mathematics

Question

Bob invests 600 euros in a bank that offers a rate of 2.75% compounded annually.

- Calculate how much money Bob has in the bank after 4 years.
- Calculate the number of years it will take for the investment to double.

```
Compound Interest:Ban
n = 4
I% = 2.75
PU = -600
PMT = 0
FV = 668.772556
P/Y = 1
↓
| n | I% | PV | PMT | FV | AMT |
|---|---|---|---|---|---|
```

```
Compound Interest:Ban
n = 26.551355862
I% = 2.75
PU = -600
PMT = 0
FV = 1200
P/Y = 1
↓
| n | I% | PV | PMT | FV | AMT |
|---|---|---|---|---|---|
```

(a)

Student's answer:

$$600(1 + 2.75/100)^4 = 668.77.$$

(b)

Student's answer:

$$600(1 + 2.75/100)^n = 1200$$

$$n = 26.$$

Question

Ann invests 600 euros in another bank that offers interest compounded annually. Her investment doubles in 20 years. Find the rate that the bank is offering.

```
Compound Interest:Ban
n = 20
I% = 3.526492384
PU = -600
PMT = 0
FV = 1200
P/Y = 1
↓
| n | I% | PV | PMT | FV | AMT |
|---|---|---|---|---|---|
```

Student's answer:

$$600(1 + r/100)^{20} = 1200$$

$$r = 3.53\%.$$

Question

A fixed amount of \$1100 is invested in a bank at the beginning of the year at an interest rate of 12% per annum, compounded monthly. How much money is in the bank account at the end of the year?

```
Compound Interest:End
n = 1
I% = 12
PU = -1100
PMT = 0
FV = 1239.507533
P/Y = 1
↓
| n | I% | PV | PMT | FV | AMT |
|---|---|---|---|---|---|
```

Student's answer:

$$1100(1 + 12/1200)^{12} = 1239.51$$

Examples taken from the bank of specimen questions

Paper 1, question 4

Bob invests 600 euros in a bank that offers a rate of 2.75% compounded annually.

- Calculate how much money Bob has in the bank after 4 years.
- Calculate the number of years it will take for the investment to double.

Ann invests 600 euros in another bank that offers interest compounded annually. Her investment doubles in 20 years.

- Find the rate that the bank is offering.

The student should use the calculator mode and then either "TABLE" or "TVM".

(a)

- Input $Y1 = 600(1 + 2.75/100)^x$.

%	Y1
3.8	665.15
3.9	666.96
4	668.77
4.1	670.58

4

FORM DEL ROW G-COM G-PLT

Compound Interest:End					
n	=	4			
I%	=	2.75			
PV	=	-600			
PMT	=	0			
FV	=	668.7727556			
P/Y	=	1			
n	I%	PV	PMT	FV	AMT

Student's answer:

$$600(1 + 2.75/100)^4 = 668.77.$$

(b)

- Scroll down the table until the value 1200 appears for y .

%	Y1
25.5	1198.3
25.6	1201.6
25.7	1204.8
25.8	1208.1

25.6

FORM DEL ROW G-COM G-PLT

Compound Interest:End					
n	=	25.55063862			
I%	=	2.75			
PV	=	-600			
PMT	=	0			
FV	=	1200			
P/Y	=	1			
n	I%	PV	PMT	FV	AMT

The first value is 25.6, therefore the answer is: 26 years.

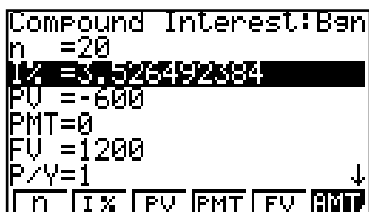
Student's answer:

$$600(1 + 2.75/100)^n = 1200$$

$$n = 26.$$

(c)

- The answer cannot be found using tables. Use "TVM".



Student's answer:

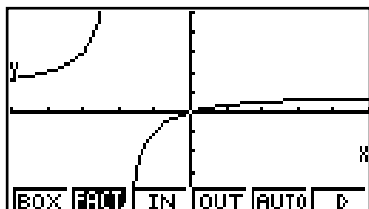
$$600(1 + r/100)^{20} = 1200$$

$$r = 3.53\%.$$

Paper 1, question 11

- (a) Sketch a graph of $y = \frac{x}{2+x}$ for $-10 \leq x \leq 10$.
- (b) Hence write down the equations of the horizontal and vertical asymptotes.

(a)

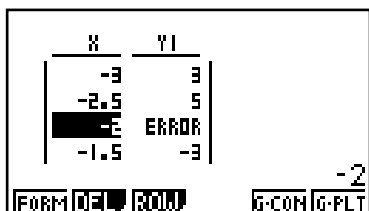


(b)

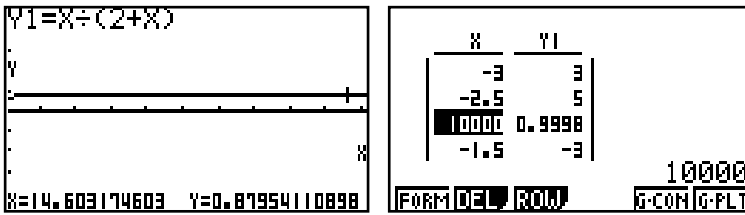
- To find the vertical asymptote the student can use the trace function on the graph or use the table of values (where "ERROR" appears in the y -value, then the corresponding value of x is where the vertical asymptote is).

Student's answer:

The vertical asymptote is at $x = -2$.



- To find the horizontal asymptote, the student will have to trace the curve for large or small x -values, or enter large x into the table.



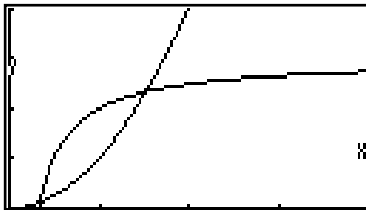
Student's answer:

The horizontal asymptote will be when $y = 1$.

Paper 2, question 1

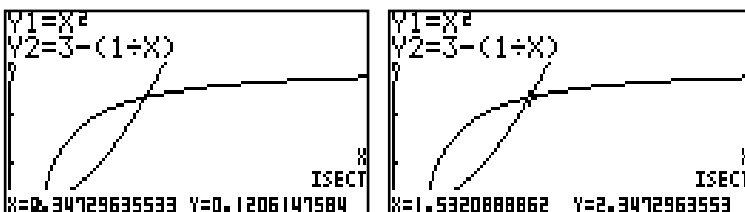
- (a) On the same graph sketch the curves $y = x^2$ and $y = 3 - \frac{1}{x}$ for values of x from 0 to 4 and values of y from 0 to 4. Show your scales on your axes.
- (b) Find the points of intersection of these two curves.
- (c) (i) Find the gradient of the curve $y = 3 - \frac{1}{x}$ in terms of x .
 (ii) Find the value of this gradient at the point (1, 2).
- (d) Find the equation of the tangent to the curve $y = 3 - \frac{1}{x}$ at the point (1, 2).

(a)



- The student should add the scales.

(b)



(c)

Student's answer:

- (i) $dy/dx = 1/x^2$.

(ii) $dy/dx = 1/1 = 1$.

This value can also be found on the GDC.

x	y^1	y^2	y^2
0.5	1	1	4
1	2	2	
1.5	3	2.3333	0.4444
2	4	2.5	0.25

(d)

Student's answer:

$$y = mx + c$$

$$y = 1x + c$$

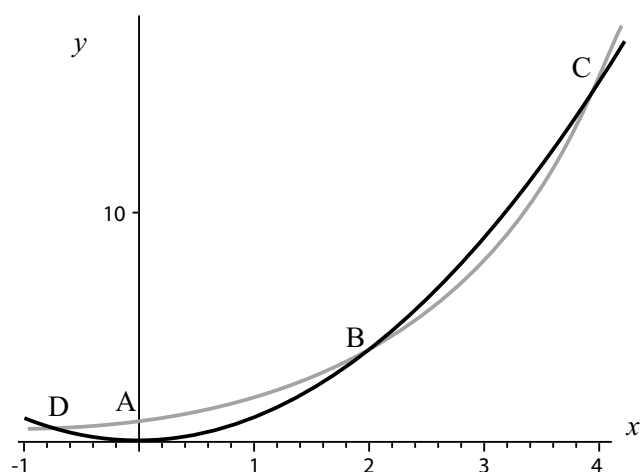
$$2 = 1 \times 1 + c$$

$$c = 1$$

$$y = x + 1.$$

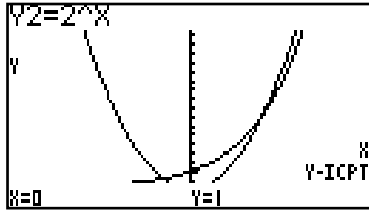
Paper 2, question 3

The figure below shows the graphs of the functions $y = x^2$ and $y = 2^x$ for values of x between -2 and 5 . The points of intersection of the two curves are marked as B, C and D.



- Write down the coordinates of the point A.
- Write down the coordinates of the points B and C.
- Find the x -coordinate of the point D.
- Write down, using interval notation, all values of x for which $2x \leq x^2$.

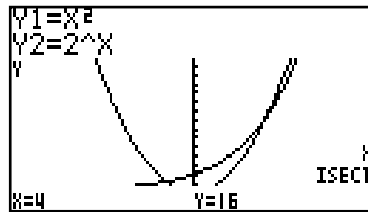
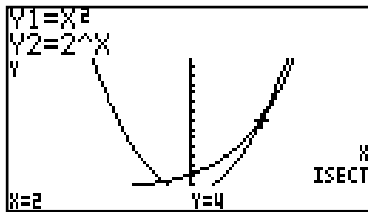
(a)



Student's answer:

When $x = 0$, $y = 1$ and so the coordinates of A are $(0, 1)$.

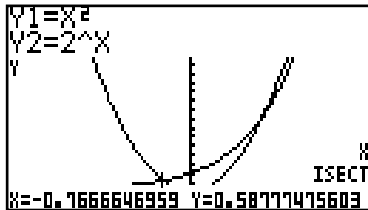
(b)



Student's answer:

The coordinates of B are $(2, 4)$ and of C are $(4, 16)$.

(c)



Student's answer:

The x -coordinate of D is -0.767 .

(d)

From the graph and the intersection points, students should be able to see that $2^x \leq x^2$ for $2 \leq x \leq 4$ and $-\infty \leq x \leq -0.767$.

Paper 2, question 4

At the circus a clown is swinging from an elastic rope. A student decides to investigate the motion of the clown. The results can be shown on the graph of the function $f(x) = (0.8^x)(5 \sin 100x)$, where x is the horizontal distance in metres.

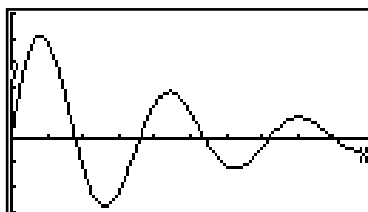
- Sketch the graph of $f(x)$ for $0 \leq x \leq 10$ and $-3 \leq f(x) \leq 5$.
- Find the coordinates of the first local maximum point.
- Find the coordinates of one point where the curve cuts the x -axis.

Another clown is fired from a cannon. The clown passes through the points given in the table below:

Horizontal distance (x)	Vertical distance (y)
0.00341	0.0102
0.0238	0.0714
0.563	1.69
1.92	5.76
3.40	10.2

- Find the correlation coefficient, r , and comment on the value for r .
- Write down the equation of the regression line of y on x .
- Sketch this line on the graph of $f(x)$ in part (a).
- Find the coordinates of one of the points where this line cuts the curve.

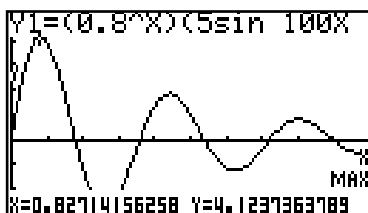
(a)



- The student should add scales to the sketch.

(b)

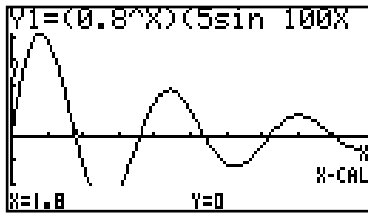
- Use "G-SOLV", then "MAX".



Student's answer:

The first maximum point is at $(0.827, 4.12)$.

(c)



Student's answer:

(1.8, 0).

(d)



Student's answer:

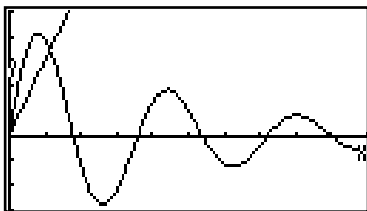
$r = 1$, therefore the correlation is perfect and positive.

(e)

Student's answer:

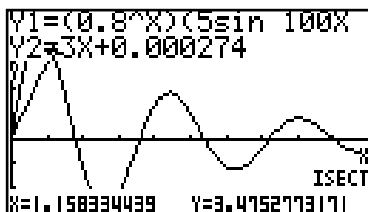
The equation of the regression line is $y = 3x + 0.000274$ or $y = 3x$.

(f)



- The student should add scales to the sketch.

(g)



Student's answer:

One point of intersection is (1.16, 3.48).

Main features

Run mode

Press "Set up" on the GDC.

```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Func Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```

- "Scientific (Sci)" is useful for standard form. Students must be able to transfer this notation back to mathematical notation.
- "Float" gives the number of figures that occur after the decimal point.
- Students should check that the GDC is in "Degree" mode.
- Students should check that the GDC is in "Func" mode.
- "Connected" draws graphs as continuous functions or as dots.
- "Sequential" draws graphs one by one or simultaneously.
- "Real" is the mode for mathematical studies students.
- With "Full" the graph uses the full screen.
- With "Horiz" the graph takes up the top half of the screen. Students can do calculations on the bottom half.
- "G-T" splits the screen vertically. The left side is the graph and the right side is the table of values.

```
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1
```

- "WINDOW": here the user enters the required x - and y -values.

```
ZOOM MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7↓ZTrig
```

ZOOM

- “4: ZDecimal” is useful in removing vertical asymptotes from graphs.
- “6: ZStandard” gives the scale from -10 to 10 .
- “7: ZTrig” is useful for trigonometric functions.
- “0: ZoomFit” will fit the graph on to the screen.

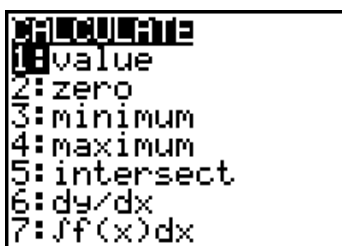


FORMAT (2nd ZOOM)

- The screen shows if axes are on, etc.

Press “TRACE” on the GDC to move the cursor along the graph.

Use “CALCULATE (2nd Trace)” to carry out various calculations.



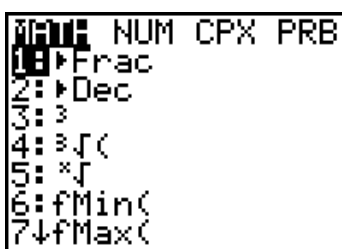
- “1:value” calculates the value of y for any given x -value.
- “2:zero” calculates the intersection with the x -axis. Place the cursor to the left of the intersection point, press “ENTER”, then place the cursor to the right of the intersection point, press “ENTER”. Press “ENTER” once more to display zero value.
- “3:minimum” calculates the minimum turning point—as above for zero values.
- “4:maximum” calculates the maximum turning point—as above for zero values.
- “5:intersect” calculates where two curves intersect. Place the cursor near the intersection point and press “ENTER” three times.
- “6:dy/dx” gives the value of the gradient for the x -value that the student types in.
- “7:∫f(x)dx” gives the area under the curve between the two values of x that the user enters.

Graph mode

Press “GRAPH” to draw the graph of function(s) entered under “y = ”.

Table mode

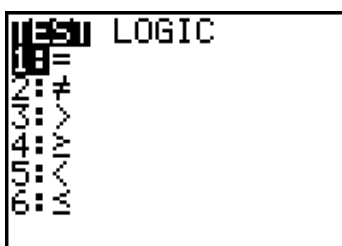
In this mode the GDC displays the table of values for the equation entered in “y =”.



“MATH” view

- “1:” gives the answer as a fraction.
- “2:” gives the answer as a decimal.
- “3:” gives the cube of a number (for example, the student inputs “10, Math, 3, enter”).
- “4:” gives the cube root.
- “5:” gives any root.
- “6:” gives the x -value of the minimum.
- “7:” gives the x -value of the maximum.
- “8:” gives the value of the derivative at a certain point (the student inputs “nDeriv(function, x , value)ENTER”).
- “9:” gives the area under the curve between two points (the student inputs “fnInt(function, x , lower limit, upper limit) ENTER”).
- “10:” gives the solution for an unknown (see later examples).

NUM, CPX and PRB are rarely used by mathematical studies students.



“TEST (2nd MATH)” view

- “TEST” gives signs for equal, not equal, less than, etc.
- “LOGIC” gives and, or, xor (exclusive or) and not functions (can be used to help with truth tables).

Statistics mode

```

CALC TESTS
1:Edit...
2:SortA(
3:SortD(
4:ClrList
5:SetUpEditor
  
```

```

EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg
  
```

```

EDIT CALC TESTS
1:Z-Test...
2:T-Test...
3:2-SampZTest...
4:2-SampTTest...
5:1-PropZTest...
6:2-PropZTest...
7↓ZInterval...
  
```

- “EDIT” view displays the following functions:
 - “1: Edit” (to enter values in lists)
 - “2: SortA” (sorts lists in ascending order)
 - “3: SortD” (sorts lists in descending order)
 - “4: ClrList” (clears lists)
 - “5: SetUpEditor”.
- “CALC” view displays the following functions:
 - “1: 1-Var Stats” (used for mean, etc)
 - “2: 2-Var Stats” (used in scatter graphs for the mean points and in linear regression if the mean and standard deviation of x and y are required)
 - “3: Med-med”
 - “4: LinReg($ax+b$)”
 - “5: QuadReg”, etc (not many of these functions are used in the mathematical studies course).
- “TESTS” view gives the user access to several functions. The two most commonly used in the mathematical studies course are “C: x^2 -Test” and “E: LinRegTTest”.

```

NAMES OPS MATH
1:L1
2:L2
3:L3
4:L4
5:L5
6:L6
7:RESID
  
```

```

NAMES OPS MATH
1:SortA(
2:SortD(
3:dim(
4:Fill(
5:seq(
6:cumSum(
7↓ΔList(
  
```

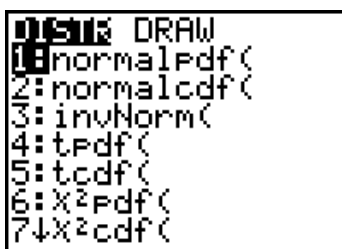
```

NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:prod(
7↓stdDev(
  
```

LIST (2nd STAT)

- “NAMES” view displays “1:List1”, etc.
- “OPS”:
 - “1: SortA” (sorts in ascending order)
 - “2: SortD” (sorts in descending order)
 - “3: dim” (gives dimensions)
 - “4: Fill”
 - “5: seq” (used for sequences)
 - etc.

- “MATH” view enables students to calculate the minimum, maximum, mean, etc of numbers in a list. The numbers must be entered, for example, as follows: Mean({2,3,4,5,6}).



DISTR (2nd VARS)

- “1: normalpdf”.
- “2: normalcdf” (to find the area under the standard normal curve).
- “3: invNorm” (to find the z score or x -value).
- etc.

Finance

Press “APPS”, “Finance”, then “CALC”, then “TVM Solver” to get the window shown below.



- “N:” gives the total number of payment periods.
- “I%:” gives the annual interest rate.
- “PV:” gives the present value, entered as a negative to represent cash outflow (investment).
- “PMT:” gives the payment amount.
- “FV:” gives the future value.
- “P/Y:” gives the number of payment periods per year.
- “C/Y:” gives the number of compound periods per year.

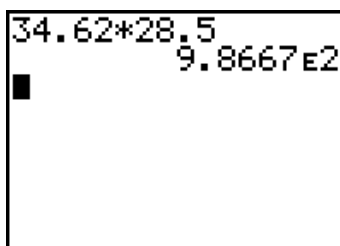
Because there are no payments when solving compound interest problems, “PMT” must be set to “0” and “P/Y” must be set to “1” (see the financial mathematics examples on pages 45–6).

Examples of the main features in use for the mathematical studies SL course

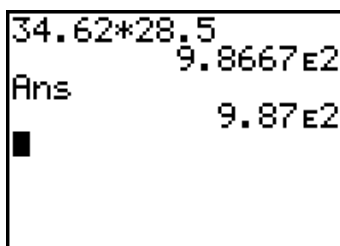
Scientific mode

Question

Calculate 34.62×28.5 and write your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.



Students must realize that $E2$ means 10^2 and write their answer using the correct mathematical terminology.



If the question asks for the answer to be given correct to two decimal places, the student's answer should be:

$$(34.62 \times 28.5) = 9.87 \times 10^2.$$

Arithmetic progression/geometric progression

Question

The first term of an arithmetic progression is 8, the common difference is 7.

- Find the 20th term.
- Find the sum of 15 terms.

(a)

- Use "LIST", then "OPS", then "5". Input "8 + 7x, x, 0, 19" and press "ENTER". This gives all terms from 8 to 141. 141 is the 20th term.

```
seq(8+7X,X,0,19)
...20 127 134 141)
```

Student's answer:

$$U_{20} = 8 + (19 \times 7) = 141.$$

(b)

- Use "LIST", then "MATH", "5", "LIST", "OPS", "5". Input "8+7x, x, 0, 14" and press "ENTER". This gives 855.

```
sum(seq(8+7X,X,0
,14))
855
```

Student's answer:

$$S_{15} = 15/2 (2 \times 8 + 14 \times 7) = 855.$$

If students are given a number and are asked to find which term it is, they can use "SOLVER".

Find which term of the sequence with 3 as first term and common ratio of 8 first exceeds 200.

- Use "MATH" then "SOLVER". To enter data, press the up arrow, fill in the information "AR(x-1) - 200" and press "ENTER".
- Fill in value for A = 3, R = 8, place cursor on X and press "ALPHA" then "ENTER" to get solution. Here $X = 3.0196 \dots$ so the 4th term would exceed 200.

```
A*R^(X-1)-200=0
A=3
R=8
▪ X=3.0196312296...
bound={-1E99,1...
▪ left-rt=0
```

Student's answer:

$$3 \times 8^{(n-1)} > 200.$$

$$n = 4.$$

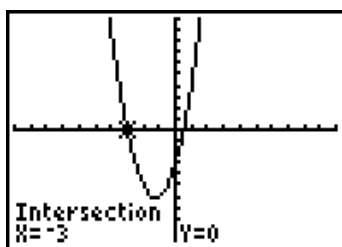
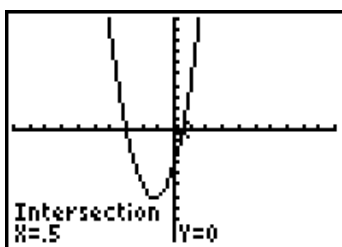
Solution of quadratics

Question

Solve for x the equation $2x^2 + 5x - 3 = 0$.

By graphing

- $Y_1 = 2x^2 + 5x - 3$.
- $Y_2 = 0$.
- Check window, then press "GRAPH".
- "2nd TRACE", then "5", then move the cursor near an intersection point. Press "ENTER" three times to get the answer. Repeat for the second intersection point.



Using "SOLVER"

- Use "MATH", then "SOLVER". To enter data, press the up arrow, enter the equation, then press "ENTER". Press "ALPHA" then "ENTER" for the answer.
- This gives one point of intersection. Put in another "guess" for x and then press "ALPHA" and "ENTER" again.

```

2X^2+5X-3=0
■ X= -3.000000000...
bound=■ -1E99, 1...
■ left-rt=0
  
```

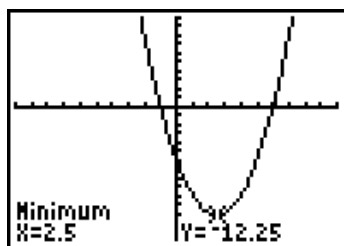
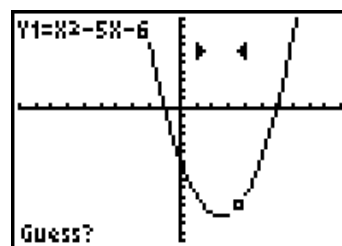
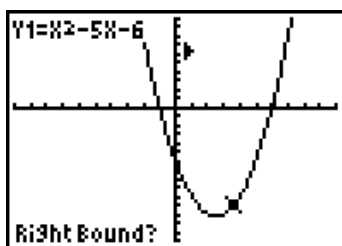
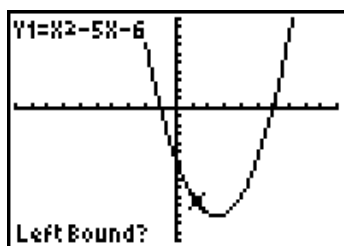
The student may write down the solutions without showing any further working. If any more is required, the question could first ask the student to factorize the function.

Vertex of quadratic

Question

Find the vertex of the function $y = x^2 - 5x - 6$.

- Input "Y =".
- Input "Y1 = $x^2 - 5x - 6$ ".
- Use "GRAPH", then "2nd TRACE", then "3". Set the cursor to the left of the vertex, then press "ENTER". Set the cursor to the right of the vertex, press "ENTER", press "ENTER" again to get the coordinates.



The student's answer could illustrate at least two possible scenarios depending on what the question-setter is aiming to test.

To show that the vertex is on the axis of symmetry, the student could write:

$$x = (-b/2a) = 2.5 \ (5/2); \ y = -12.25.$$

If differentiation is required, the student could write:

$$dy/dx = 2x - 5 = 0 \text{ at stationary values}$$

$$\Rightarrow x = 5/2; \ y = -12.25.$$

If no method is specified, then both methods are acceptable.

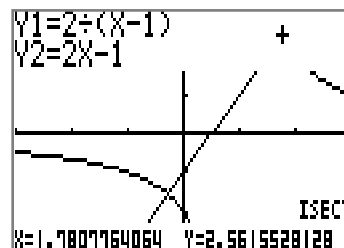
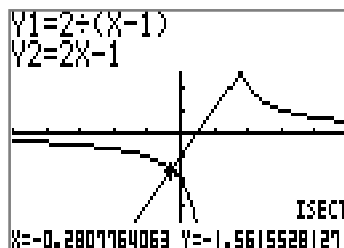
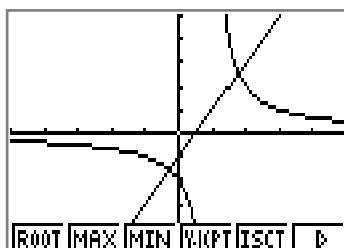
Intersection of curves

Graph the curves and use the intersection function as shown previously.

Question

Find the points of intersection of the graphs of the following functions:

$$y = 2/(x - 1) \text{ and } y = 2x - 1.$$



It is sufficient for the student to write:

$$2/(x - 1) = 2x - 1 \Rightarrow x = -0.281 \text{ and } 1.78.$$

Exponential equations

Using "Graph"

"Graph": $Y_1 = a^x$.

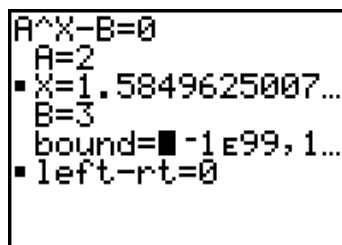
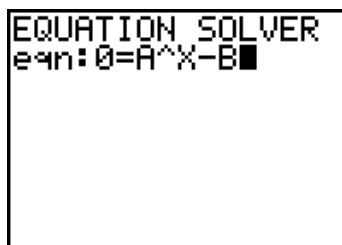
"Graph": $Y_2 = b$.

Then use the intersection function as used previously.

OR Using "SOLVER"

Press "MATH", then "SOLVER" and "ENTER". Press the up arrow, enter the equation " $A^x - B$ " and press "ENTER". Fill in values for A and B, then place the cursor at X and press "ALPHA ENTER" for the solution.

Find the value of x for the equation above if $A = 2$ and $B = 3$.



Student's answer:

$$2^x = 3 \Rightarrow x = 1.58 \text{ (no working need be shown).}$$

Simultaneous equations

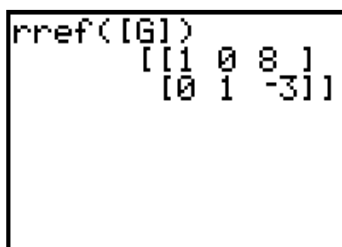
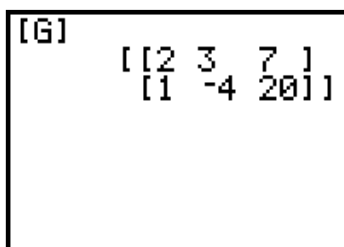
Using matrices

Question

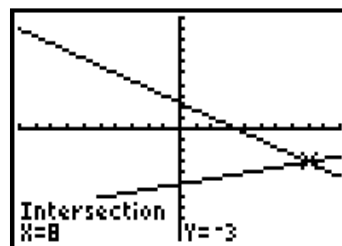
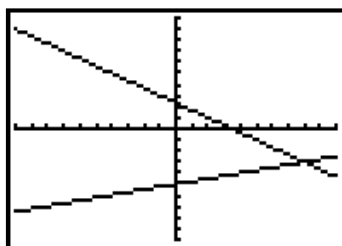
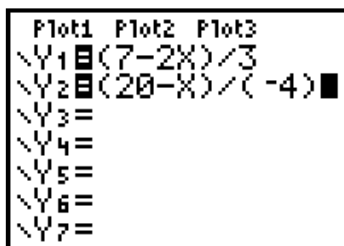
Solve the equations $2x + 3y = 7$
 $x - 4y = 20.$

- Use: "MATRIX - EDIT - ENTER - 2 - ENTER - 3 - ENTER - 2 - ENTER - 3 - ENTER - 7 - ENTER - 1 - ENTER - -4 - ENTER - 20 - ENTER - 2ND MODE (QUIT) - MATRIX - MATH - B: rref - ENTER - MATRIX - ENTER - ENTER".

The answer is $x = 8$ and $y = -3.$



- Alternatively, the student can rearrange the equations to $y = (7 - 2x)/3$ and $y = (20 - x)/(-4)$, then graph and find the intersection point.



Students should write down both equations and give the solutions for x and y .

Truth tables

Using "1" for "True" and "0" for "False" enter values in lists for p and q .

Question

Complete the truth table for $(p \vee q) \Rightarrow r.$

- Enter (11110000) in List 1, (11001100) in List 2 and (10101010) in List 3.
- Then use "Test Logic - not L1 - Test Logic - xor - ENTER - L2) - Test Logic - or - L3 - ENTER".

L1	L2	L3	3
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

L3(9) =			

```
not(L1 xor L2) o
r L3
(1 1 1 0 1 0 1 ...
```

- Alternatively, the student can input this equation as List 4.

L2	L3	L4	4
1	1	1	
1	0	1	
0	1	1	
0	0	0	
1	1	1	
1	0	0	
0	1	1	

L4 = {1, 1, 1, 0, 1, 0...			

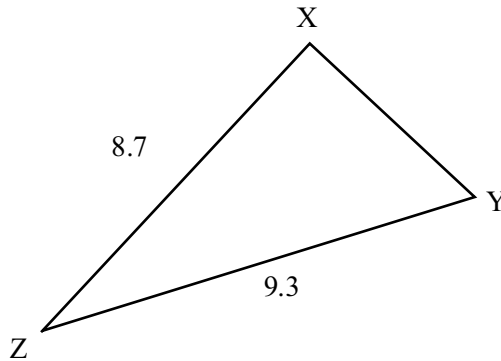
Note that $p \Rightarrow q$ is the same as $\neg p \vee q$.

Students should show all the intermediate steps even though the final answer can be obtained from the GDC.

Cosine rule using "SOLVER"

Question

In triangle XYZ, $XZ = 8.7$ cm, $YZ = 9.3$ cm and angle $XZY = 33^\circ$. Find the length of side XY.



- Use "MATH - SOLVER - ENTER", then press the up arrow. Enter the equation $X^2 + Y^2 - Z^2 - 2XY\cos A$, then press "ENTER". Fill in the values for X, Y and A, then put the cursor at Z and press "ALPHA ENTER" to get the answer for the side XY.
- If the student wants an angle, they can follow the same procedure but fill in the values they already know and place the cursor at the letter they want to calculate.

```
EQUATION SOLVER
eqn: 0=X^2+Y^2-Z^2-2
XYcos(A)
```

```
X^2+Y^2-Z^2-2XYc...=0
X=9.3
Y=8.7
Z=5.1445435847...
A=33
bound=-1E99,1...
left-rt=0
```

The student should put the corresponding values into the cosine rule and then write the answer, for example:

$$Z^2 = 9.3^2 + 8.7^2 - 2 \times 9.3 \times 8.7 \cos(33^\circ)$$

$$Z = 5.14.$$

Matrices

These will only be required for entering observed data in the chi-squared distribution.

- “NAMES” gives a list of the matrices the user has entered.

```
NAMES MATH EDIT
1: [A] 2x3
2: [B] 3x3
3: [C] 3x3
4: [D] 2x1
5: [E] 2x3
6: [F]
7↓ [G]
```

- “MATH” allows the user to find the determinant, transpose, dimension, etc.

```
NAMES MATH EDIT
1: det(
2: T
3: dim(
4: Fill(
5: identity(
6: randM(
7↓ augment(
```

- “EDIT” allows the user to alter the information stored, or to enter new information.

```
NAMES MATH EDIT
1: [A] 2x3
2: [B] 3x3
3: [C] 3x3
4: [D] 2x1
5: [E] 2x3
6: [F]
7↓ [G]
```

- To enter a 2×3 matrix, use “MATRIX – EDIT – 2 – ENTER – 3 – ENTER”, then put in entries.

```
MATRIX[A] 2 x3
[ 2      3      7 ]
[ 1      -4     20 ]
```

See also the example in the chi-squared section on pages 41–2.

Statistics

- “EDIT” allows the user to enter data in lists, sort the lists in ascending or descending order, and clear the lists.

```
EDIT CALC TESTS
1:Edit...
2:SortA(
3:SortD(
4:ClrList
5:SetUpEditor
```

- “CALC” allows the user to calculate various statistics.

```
EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7:QuartReg
```

- “TESTS” allows the user to perform various tests.

```
EDIT CALC TESTS
1:Z-Test...
2:T-Test...
3:2-SampZTest...
4:2-SampTTest...
5:1-PropZTest...
6:2-PropZTest...
7:ZInterval...
```

- Use “STAT – EDIT – ENTER” to put values in the lists.

L1	L2	L3	Σ
1	2	1	
2	4	1	
3	6	1	
4	10	1	
5	14	1	
6	16	4	
7	20	3	
L2(1)=2			

- In "L1" are the numbers 1,2,3,4,5,6,7.
- In "L2" are the numbers 2,4,6,10,14,16,20.
- To find the mean, median, standard deviation, etc of a list of numbers, if "L1" represents the numbers and "L2" the frequencies, then press "STAT – CALC – 1:1-Var Stats – ENTER – L1 (2nd 1) – comma (,) – L2 (2nd 2) – ENTER".

```

1-Var Stats
x̄=5.194444444
Σx=374
Σx²=2138
Sx=1.658430353
σx=1.646873206
↓n=72
    
```

- In this view, scroll down to obtain more values.

```

1-Var Stats
↑n=72
minX=1
Q1=4
Med=5.5
Q3=7
maxX=7
    
```

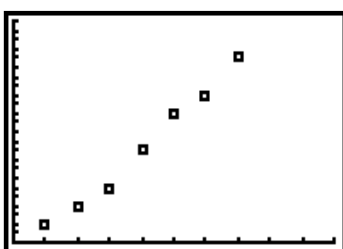
For grouped data, using the mid-point of the groups gives an estimate of the mean and standard deviation, but this does **not** give accurate values for the quartiles.

Students may write down the values of the mean and standard deviation from the calculator without showing any further working, but they must be careful to enter the data correctly. A wrong answer from the GDC with no method shown is awarded no marks.

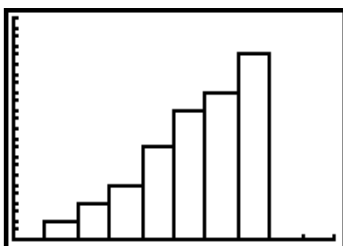
Scatter diagram, histogram, box plot, cumulative frequency

Press "STAT PLOT – ENTER – plot1 on – ENTER", then scroll down to the type of graph that the student wants to display. Fill in the lists that the data is in, set the window view, then press "Graph".

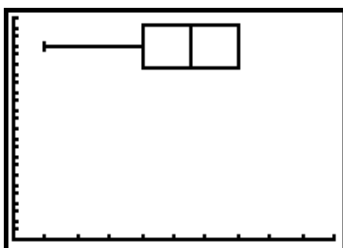
- For the data in lists 1 and 2 above, the following scatter graph is obtained (students should also put scales on their graph).



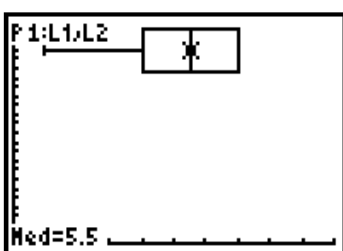
- The following bar chart is obtained (scales must be entered on the axes).



- The following box plot is obtained (scales must be put on the axes).

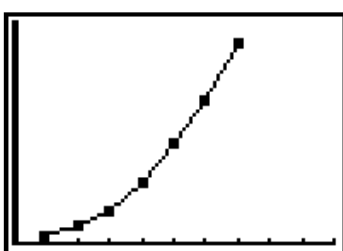


- By pressing the "TRACE" key, the student can read off the main features on the box plot (scales must be put on the axes).



For cumulative frequency table, move the cursor to highlight "L3 – LIST – OPS – CumSum – ENTER – L2 – ENTER". The cumulative frequencies appear in list 3.

Use "StatPlot", then highlight "Type 2 graph". The xList is L1 and the yList is L3. Make sure that the window is suitable, then press "Graph".



Chi-squared test

Enter observed data in matrix. In the example the observed data is

5	6	12
8	10	8
9	9	14

entered in matrix B.

- Use "STAT – TESTS – C: χ^2 – Test – ENTER". Put [B] after "Observed" and [C] after "Expected" (go to "MATRIX" to enter these letters). Put the cursor on "Calculate" and press "ENTER" for the chi-squared value and degrees of freedom. The calculator works out the expected values automatically.

```

 $\chi^2$ -Test
Observed: [B]
Expected: [C]
Calculate Draw
    
```

```

[C]
[[6.25 7.10 9.6...
 [7.06 8.02 10....
 [8.69 9.88 13....
    
```

```

 $\chi^2$ -Test
 $\chi^2=2.490374611$ 
 $p=.6463601936$ 
 $df=4$ 
    
```

Unless another type of question is set, this value for chi-squared, the degrees of freedom and the expected values can all be obtained from the GDC. However, students should be familiar with the formula and know how to use it.

Regression line

- Enter data in "L1" and "L2" (or any of the other lists).
- Use "STAT – TESTS – E: LinRegTTest – ENTER", then enter the names of the lists the student has used. Move the cursor down to "Calculate", then press "ENTER".
- The regression line is obtained and the correlation coefficient, r .

```

LinRegTTest
Xlist:L1
Ylist:L2
Freq:1
 $\beta$  &  $\rho$ :  $\neq$  <0 >0
RegEQ:
Calculate
    
```

```

LinRegTTest
 $y=a+bx$ 
 $\beta \neq 0$  and  $\rho \neq 0$ 
 $t=20.04885338$ 
 $p=5.706227E-6$ 
 $df=5$ 
 $\downarrow a=-2$ 
    
```

```

LinRegTTest
 $y=a+bx$ 
 $\beta \neq 0$  and  $\rho \neq 0$ 
 $\uparrow b=3.071428571$ 
 $s=.8106434834$ 
 $r^2=.9877136752$ 
 $r=.9938378516$ 
    
```

Alternatively, use "STAT – CALC LinReg($ax + b$)". However, before using this calculation, "DiagnosticOn" must be set (this is found in "CATALOG").

```

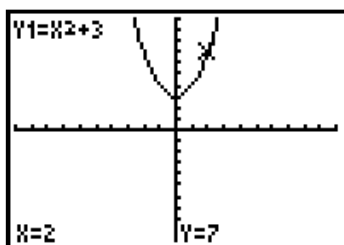
LinReg
 $y=ax+b$ 
 $a=3.071428571$ 
 $b=-2$ 
 $r^2=.9877136752$ 
 $r=.9938378516$ 
    
```

If data are provided, then the students should be able to write down the correlation coefficient, r , and the equation of the regression line directly from their GDC. However, they should also be familiar with the formulae and how to use them.

Calculus

To calculate the value of Y at any point on a curve

Draw the curve, say $y = x^2 + 3$, go to "CALC - value", then enter the value of x , say $x = 2$, then press "ENTER".



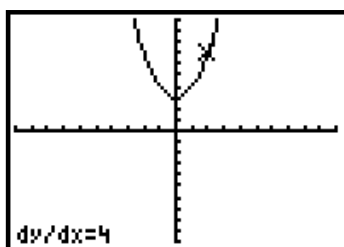
- "Table" can also be used to find this value.

X	Y1
0	3
1	4
2	7
3	12
4	19
5	28
6	39

X=0

To calculate the value of the derivative at any point X on a curve

Draw the curve, say $y = x^2 + 3$, go to "CALC - dy/dx ", then enter the value of x , say $x = 2$, then press "ENTER".



Student's answer:

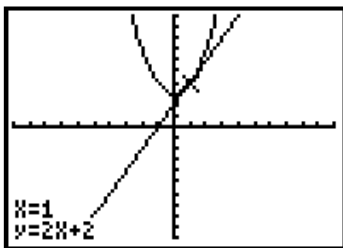
$$dy/dx = 2x$$

when $x = 2$, $dy/dx = 4$.

Equation of tangent line

Enter the equation in $Y1 =$. Use "Graph - ENTER - DRAW - 5:Tangent - ENTER". Put in the value of x where the student has to find the equation of the tangent, then press "ENTER". The tangent line is drawn and its equation is given.

Find the equation of the tangent to the curve $y = x^2 + 3$ at the point $(1, 4)$.



Student's answer:

$$dy/dx = 2x$$

$$\text{at } x = 1, dy/dx = 2 (=m)$$

$$y = 2x + c$$

$$4 = 2 \times 1 + c$$

$$c = 2.$$

The equation is $y = 2x + 2$.

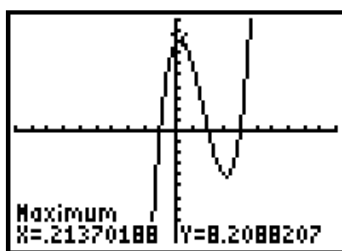
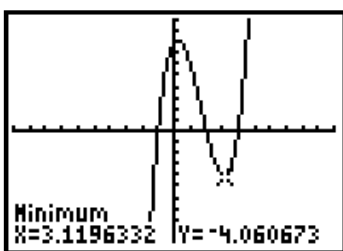
Finding local maximum and minimum points

Question

Find the local maximum and minimum points on the graph of the curve

$$y = x^3 - 5x^2 + 2x + 8.$$

- Enter the equation in $Y1 =$. Then use "Graph - CALC - minimum - enter". Put the cursor to the left of the minimum point, then press "ENTER". Put the cursor to the right of the minimum point, press "ENTER" and then press "ENTER" again. Repeat the process for the maximum point.



Student's answer:

$$dy/dx = 3x^2 - 10x + 2 = 0 \text{ at stationary values.}$$

(Students will then have to solve the quadratic using a programme, the solver function, by graphing or by using the quadratic formula.)

Student's answer:

$$x = 3.12 \text{ or } 0.214.$$

(If students enter the derived equation into $Y2$, they can then use "Calc Value" or "Table" to find the y -values.)

Financial mathematics

Question

Bob invests 600 euros in a bank that offers a rate of 2.75% compounded annually.

- Calculate how much money Bob has in the bank after 4 years.
- Calculate the number of years it will take for the investment to double.

(a)

```

N=4
I%=2.75
PV=-600
PMT=0
▪ FV=668.7727556
P/Y=1
C/Y=1
PMT: [ ] BEGIN
    
```

Student's answer:

$$600(1 + 2.75/100)^4 = 668.77.$$

(b)

```

▪ N=25.55035862
I%=2.75
PV=-600
PMT=0
FV=1200
P/Y=1
C/Y=1
PMT: [ ] BEGIN
    
```

Student's answer:

$$600(1 + 2.75/100)^n = 1200$$

$$n = 26.$$

Question

Ann invests 600 euros in another bank that offers interest compounded annually. Her investment doubles in 20 years. Find the rate that the bank is offering.

```

N=20
▪ I%=3.526492384
PV=-600
PMT=0
FV=1200
P/Y=1
C/Y=1
PMT: [ ] BEGIN
    
```

Student's answer:

$$600(1 + r/100)^{20} = 1200$$

$$r = 3.53\%.$$

Question

A fixed amount of \$1100 is invested in a bank at the beginning of the year at an interest rate of 12% per annum, compounded monthly. How much money is in the bank account at the end of the year?

```
N=1
I% = 12
PV = -1100
PMT = 0
FV = 1239.507533
P/Y = 1
C/Y = 12
PMT: [ ] [ ] BEGIN
```

Student's answer:

$$1100(1 + 12/1200)^{12} = 1239.51.$$

Examples taken from the bank of specimen questions

Paper 1, question 4

Bob invests 600 euros in a bank that offers a rate of 2.75% compounded annually.

- Calculate how much money Bob has in the bank after 4 years.
- Calculate the number of years it will take for the investment to double.

Ann invests 600 euros in another bank that offers interest compounded annually. Her investment doubles in 20 years.

- Find the rate that the bank is offering.

The student should use the calculator mode and either the table of values or "Finance".

(a)

- Input $Y1 = 600(1 + 2.75/100)^x$.

X	Y1	
3.8	665.15	
3.9	666.96	
4	668.77	
4.1	670.59	
4.2	672.41	
4.3	674.24	
4.4	676.07	
X=4		

N=4
I%=2.75
PV=-600
PMT=0
FV=668.7727556
P/Y=1
C/Y=1
PMT: [] [] [] BEGIN

Student's answer:

$$600(1 + 2.75/100)^4 = 668.77.$$

(b)

- Scroll down the table until the value 1200 appears for y.

X	Y1	
25.5	1198.4	
25.6	1201.6	
25.7	1204.9	
25.8	1208.2	
25.9	1211.4	
26	1214.7	
26.1	1218	
X=25.6		

N=25.55035862
I%=2.75
PV=-600
PMT=0
FV=1200
P/Y=1
C/Y=1
PMT: [] [] [] BEGIN

The first value is 25.6, therefore the answer is: 26 years.

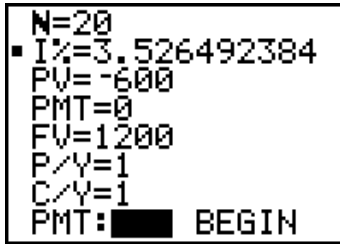
Student's answer:

$$600(1 + 2.75/100)^n = 1200$$

$$n = 26.$$

(c)

- The answer cannot be found using tables. The student must use "Finance".



Student's answer:

$$600(1 + r/100)^{20} = 1200$$

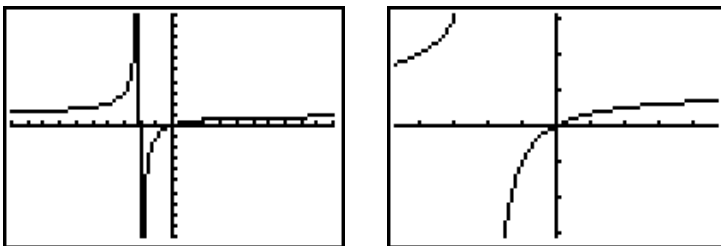
$$r = 3.53\%.$$

Paper 1, question 11

- (a) Sketch a graph of $y = \frac{x}{2+x}$ for $-10 \leq x \leq 10$.
- (b) Hence write down the equations of the horizontal and vertical asymptotes.

(a)

- The student must realize that the vertical line should not be there, and use "Zdecimal" to omit it.



(b)

- To find the vertical asymptote the student can use the trace function on the graph or use the table of values (where "ERROR" appears in the y -value, then the corresponding value of x is where the vertical asymptote is).

X	Y1
-5	1.6667
-4	2
-3	3
-2	ERROR
-1	-1
0	0
1	.33333

X = -2

Student's answer:

The vertical asymptote is at $x = -2$.

- To find the horizontal asymptote, the student will have to trace the curve for large or small x -values.

Student's answer:

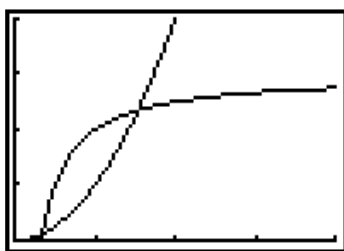
The horizontal asymptote will be when $y = 1$.

Teachers can, of course, teach the students how to calculate the asymptotes.

Paper 2, question 1

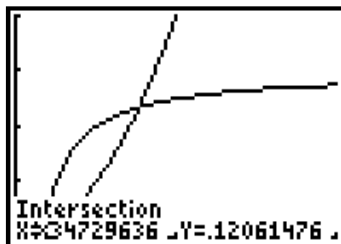
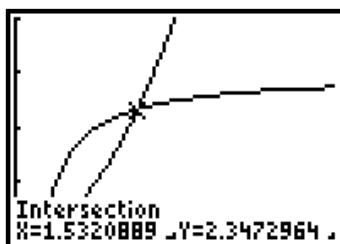
- (a) On the same graph sketch the curves $y = x^2$ and $y = 3 - \frac{1}{x}$ for values of x from 0 to 4 and values of y from 0 to 4. Show your scales on your axes.
- (b) Find the points of intersection of these two curves.
- (c) (i) Find the gradient of the curve $y = 3 - \frac{1}{x}$ in terms of x .
- (ii) Find the value of this gradient at the point $(1, 2)$.
- (d) Find the equation of the tangent to the curve $y = 3 - \frac{1}{x}$ at the point $(1, 2)$.

(a)



- The student should add the scales.

(b)



(c)

Student's answer:

(i) $dy/dx = 1/x^2$.

(ii) $dy/dx = 1/1 = 1$.

This value can also be found on the GDC.



(d)

Student's answer:

$$y = mx + c$$

$$y = 1x + c$$

$$2 = 1 \times 1 + c$$

$$c = 1$$

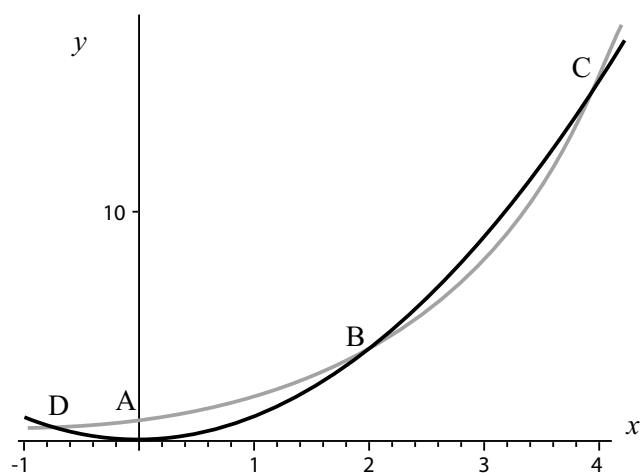
$$y = x + 1.$$

- The student should check on the GDC.



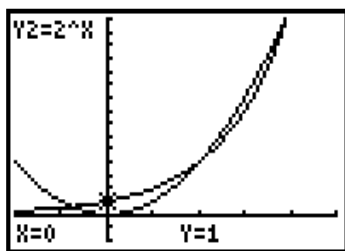
Paper 2, question 3

The figure below shows the graphs of the functions $y = x^2$ and $y = 2^x$ for values of x between -2 and 5 . The points of intersection of the two curves are marked as B, C and D.



- Write down the coordinates of the point A.
- Write down the coordinates of the points B and C.
- Find the x -coordinate of the point D.
- Write down, using interval notation, all values of x for which $2x \leq x^2$.

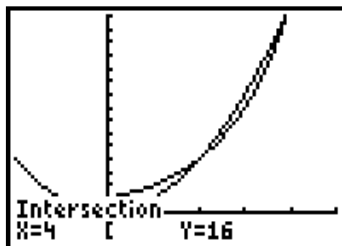
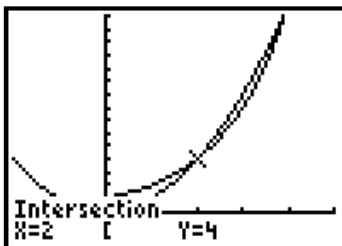
(a)



Student's answer:

When $x = 0$, $y = 1$ and so the coordinates of A are $(0, 1)$.

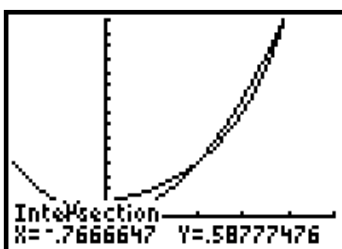
(b)



Student's answer:

The coordinates of B are (2, 4) and of C are (4, 16).

(c)



Student's answer:

The x-coordinate of D is -0.767 .

(d)

From the graph and the intersection points, students should be able to see that $2x \leq x^2$ for $2 \leq x \leq 4$ and $-\infty \leq x \leq -0.767$.

Paper 2, question 4

At the circus a clown is swinging from an elastic rope. A student decides to investigate the motion of the clown. The results can be shown on the graph of the function $f(x) = (0.8^x)(5 \sin 100x)$, where x is the horizontal distance in metres.

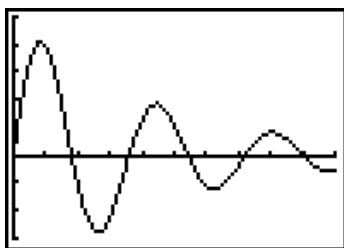
- Sketch the graph of $f(x)$ for $0 \leq x \leq 10$ and $-3 \leq f(x) \leq 5$.
- Find the coordinates of the first local maximum point.
- Find the coordinates of one point where the curve cuts the x -axis.

Another clown is fired from a cannon. The clown passes through the points given in the table below:

Horizontal distance (x)	Vertical distance (y)
0.00341	0.0102
0.0238	0.0714
0.563	1.69
1.92	5.76
3.40	10.2

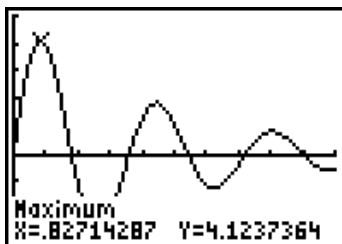
- Find the correlation coefficient, r , and comment on the value for r .
- Write down the equation of the regression line of y on x .
- Sketch this line on the graph of $f(x)$ in part (a).
- Find the coordinates of one of the points where this line cuts the curve.

(a)



- The student should add scales to the sketch.

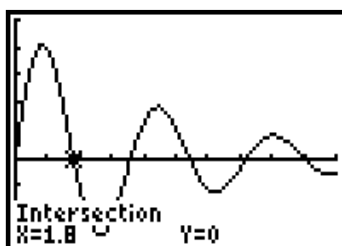
(b)



Student's answer:

The first maximum point is at (0.827, 4.12).

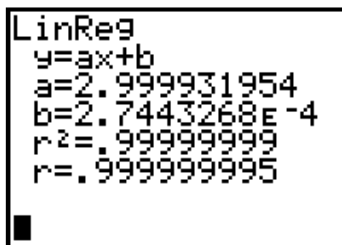
(c)



Student's answer:

(1.8, 0).

(d)



Student's answer:

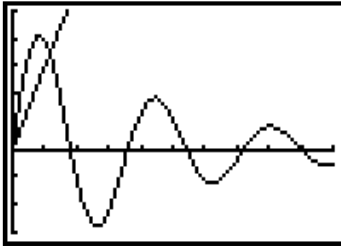
$r = 1$, therefore the correlation is perfect and positive.

(e)

Student's answer:

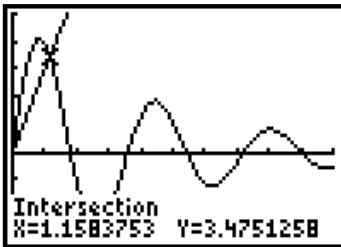
The equation of the regression line is $y = 3x + 0.000274$ or $y = 3x$.

(f)



- The student should add scales to the sketch.

(g)



Student's answer:

One point of intersection is (1.16, 3.48).

