Q10. Option A:

$$A = 10000\left(1+\frac{6}{100}\right)^6$$
$$= 14185.19.$$

Option B:

$$A = 10000\left(1+\frac{5}{100}\right)^2\left(1+\frac{6}{100}\right)^2\left(1+\frac{7}{100}\right)^2$$
$$= 14182.67.$$

(a) Option A

(b) By $14185.19 - 14182.67 = \$2.52$

Q11. First 5 yrs: $r = \frac{9}{2} = 4.5$ with $5\times 2 = 10$ periods.

For next 7 yrs: $r = \frac{8}{2} = 4$ with $7\times 2 = 14$ periods

$$\therefore A = 4000\left(1+\frac{4.5}{100}\right)^{10}\left(1+\frac{4}{100}\right)^{14}$$
$$= 10,756.96$$

i.e., \$10,756.96.

Q12. (a) $1+r_e = \left(1+\frac{10/2}{100}\right)^2$

$$\therefore r_e = 1.05^2 - 1$$
$$= 0.1025.$$

i.e., effective rate is 10.25%.

(b) $1+r_e = \left(1+\frac{10}{4}{100}\right)^4$

$$\therefore r_e = 1.025^4 - 1$$
$$= 0.1038$$

i.e., effective rate = 10.38%.

(c) $1+r_e = \left(1+\frac{10/12}{100}\right)^{12}$

$$\therefore r_e = 1.0083^{12} - 1$$
$$= 0.1047$$

i.e., effective rate = 10.47%.

Q13.

1st 10 yrs: $r = \frac{12}{4} = 3$ per quarter with $10\times 4 = 40$ quarters.

Next 12 yrs. $r = \frac{10}{2} = 5$ per half year with $12\times 2 = 24$ half yrs.

$$A = 250000\left(1+\frac{3}{100}\right)^{40}\left(1+\frac{5}{100}\right)^{24}$$
$$= 2630099.475$$

i.e., there is \$2,630,100 in the fund.

EXERCISES 21.2.4

Q1. (a) $r=8$, $C=2000$, $A=3500$

$$A = C\left(1+\frac{r}{100}\right)^n$$

$$\Leftrightarrow 3500 = 2000\,(1.08)^n$$

$$\Leftrightarrow (1.08)^n = 1.75$$

Then, using the Solve function on the graphics calc:

Solve$((1.08)\wedge X - 1.75, X, 2)$

we have, $n = 7.2714$

i.e. approx 7 yrs & 3 months

(b) $r = \frac{8}{4} = 2$, n is measured in quarters

$$\therefore 3500 = 2000\left(1+\frac{2}{100}\right)^n$$

$$\Leftrightarrow 1.75 = 1.02^n$$

$$n = 28.2597$$

$\therefore$ 28.2597 quarters $= 7.064$

i.e. approx 7.06 yrs.

(c) daily $\Rightarrow r = \frac{8}{365}$

So, $3500 = 2000\left(1+\frac{\frac{8}{365}}{100}\right)^n$

$$1.75 = (1.000219)^n$$

$$\therefore n = 2555.6$$

$\therefore$ 2555.6 days $= 7.001$ yrs.

i.e. 7 yrs.

Q2. $A = 4090$, $C = 3000$

$r = \frac{7}{4} = 1.75$ (1 period = quarter)

$$4090 = 3000\,(1.0175)^n$$

$$1.3633 = (1.0175)^n$$

$$\therefore n = 17.865$$

i.e. 17.865 quarters $= 4.466$ yrs.

i.e. (approx) $n = 18$, $4\frac{1}{2}$ yrs.

Q3. (a) $2400 = 1200\,(1.15)^n$

$$\therefore 2 = 1.15^n$$

$$\Leftrightarrow n = 4.9594$$

i.e. approx 5 yrs.

(b) $r = \frac{15}{4} = 3.75$

each period = 1 quarter.

$$2400 = 1200\,(1.0375)^n$$

$$2 = (1.0375)^n$$

$$\therefore n = 18.8284$$

i.e. 4.7070 yrs = 4 yrs 8.48 months

$\simeq$ 4 yrs 9 months

$\therefore n = 19$ quarters (approx)

= 4 yrs 9 months (approx).

Q4. $C = 1500$, $A = 1500 + 600$

$= 2100$

$r = \frac{12}{26} = 0.4615$

each period = a fortnight

$$2100 = 1500\,(1.004615)^n$$

$$1.4 = (1.004615)^n$$

$$n = 73.0765$$

i.e. 73 fortnights.

Q5. $r = \frac{7.8}{12} = 0.65$, $A = 8500$, $C = 6350$

$$\therefore 8500 = 6350\,(1.0065)^n$$

$$(1.3386) = (1.0065)^n$$

$$\therefore n = 45.0089$$

i.e. 45 months

or $\frac{45}{12} = 3.75$ yrs

= 3 yrs 9 months

Q6. (a) $C = 6000$, $r = \frac{8.5}{4} = 2.125$

5 yrs $= 5 \times 4 = 20$ quarters

$\therefore A = 6000\left(1 + \frac{2.125}{100}\right)^{20}$

$= 9136.7689$

i.e, Donald has \$9136.7689

$\therefore$ there is a shortfall

The shortfall is \$863.23

(b) i. With A now set at 12,000

we have $12000 = 6000\left(1 + \frac{2.125}{100}\right)^n$

$\Leftrightarrow 2 = (1.02125)^n$

ii. Solving for n:

$n = 32.964$

i.e, 8.241 yrs $\simeq$ 8yrs 3months

Q7. Kristian: $C = 8000$, $r = 8.4$

So, $12500 = 8000\left(1 + \frac{8.4}{100}\right)^n$

$1.5625 = (1.084)^n$

$\therefore n = 5.5331$

Jørgi: $C = 7000$, $r = \frac{9.2}{4} = 2.3$

So, $12500 = 7000\left(1 + \frac{2.3}{100}\right)^n$

$1.7857 = (1.023)^n$

$\therefore n = 25.498$

i.e, 25.498 quarters

$= 6.37$ yrs.

$\therefore$ Kristian will go first.

Time diff $= 6.3745 - 5.5331$

$= 0.8414$ yrs

$= 10$ months.

NB: If Kristian & Jørgi cannot take their money out of their account until the end of a full term, i.e, for Kristian at the end of a full year and Jørgi at the end of a 3month period, then for Kristian it would take 5.53yrs $\Rightarrow$ needs to leave money in account for 6yrs.

for Jørgi it took 25.498 quarters

$\Rightarrow$ need to leave money in account for 26 quarters or 6yrs and 6months.

i.e, Kristian would leave 6months before Jørgi under these conditions