(d) $A = 30000 \times \left(1+\frac{12}{100}\right)^{4/12}$

$= 31154.96$

i.e., approx 31155 yen

(e) $A = 28000\left(1+\frac{8}{100}\right)^{(6/12)}$

$= 29098.45$

i.e., approx 29098 yen.

(f) $A = 1600\left(1+\frac{4}{100}\right)^{(3/12)}$

$= 1615.76$

i.e., approx 1616 yen.

Q5. $P = 1500$, period $= 5$ yrs.

Account A: $r = 10$ (p.annum).

$\therefore A = 1500\left(1+\frac{10}{100}\right)^5$

$= 2415.765.$

Account B: $r = \frac{9.7}{12}$ (per month)

5yrs = 5×12 months = 60 months

$\therefore A = 1500\left(1+\frac{\frac{9.7}{12}}{100}\right)^{60}$

$= 2431.517$

Account C: daily rate $= \frac{9.5}{365}$

5yrs = 5×365 days = 1825 days.

$\therefore A = 1500\left(1+\frac{9.5/365}{100}\right)^{1825}$

$= 2411.872$

∴ Better option is B, as it returns more money.

Q6. Invest \$3000 —2yrs→ \$4000.

Compounded quarterly over 2 yrs

$\Rightarrow 4\times 2 = 8$ quarters

Let the annual rate be r% p.a

$\therefore$ rate per quarter $= \frac{r}{4}$

So, $4000 = 3000\left(1+\frac{r/4}{100}\right)^8$

$\frac{4}{3} = \left(1+\frac{r}{400}\right)^8$

$\therefore \left(\frac{4}{3}\right)^{1/8} = 1+\frac{r}{400}$

$\Leftrightarrow \frac{r}{400} = 0.03661$

$\Leftrightarrow r = 14.6458$

i.e., approx rate p.a is 14.65%.

Q7. \$7245 —5yrs→ A

For 2.5yrs, $r = 8.30$ p.a

For next 2.5yrs, $r = 9.50$ p.a

1st 2.5 yrs: Interest paid half yearly $\therefore r = \frac{8.30}{2}$ over 5 terms.

$\therefore A_1 = 7245 \times \left(1+\frac{4.15}{100}\right)^5$

$= 8878.40$

2nd 2.5 yrs: Interest paid half yearly $\therefore r = \frac{9.50}{2}$ over 5 periods.

$A_2 = 8878.40 \times \left(1+\frac{4.75}{100}\right)^5$

$= 11197.08$

$\therefore$ Interest $= \$11197.08 - 7245$

$= \$3952.08$

EXERCISES 21.2.3

Q1. (a) i. $A = 5000\left(1+\frac{6.5}{100}\right)^4$

$= 6432.33$

$\therefore$ interest $= 6432.33 - 5000$

$= 1432.33$

i.e, \$1432.33

ii. Balance $= \$6432.33$

(b) We need the interest to also be 1432.33

$\therefore \quad 1432.33 = nP \times \frac{r}{100}$

So, $1432.33 = 4 \times 5000 \times \frac{r}{100}$

$\Leftrightarrow \quad r = 7.1616$

i.e, simple rate is 7.162% p.a.

Q2. (a) 6% p.a compounded quartely means $\frac{6}{4} = 1.5\%$ per quarter

5 years means $5 \times 4 = 20$ quarters.

$\therefore A = 4200 \times \left(1+\frac{1.5}{100}\right)^{20}$

$= 5656.79$

i.e, \$5656.79.

(b) i. Interest gained $= 5656.79 - 4200$

$= 1456.79$

$\therefore$ at 6% p.a over 5 yrs, we have: $1456.79 = 5 \times P \times \frac{6}{100}$

$\Leftrightarrow \quad P = \frac{1456.79 \times 100}{30}$

$= 4855.97$

i.e, need to invest \$4856

ii. At 8.25% p.a:

$1456.79 = 5 \times P \times \frac{8.5}{100}$

$\Leftrightarrow \quad P = \frac{1456.79 \times 100}{5 \times 8.5}$

$= 3427.74$

i.e, need to invest \$3428.

Q3. Let the interest rate be r% p.a.

Every quarter, we have $\frac{r}{4}\%$ p.quarter

In 2 yrs there are 8 quarters

$\therefore \quad 4000 = 3000\left(1+\frac{r/4}{100}\right)^8$

$\Leftrightarrow \quad \frac{4}{3} = \left(1+\frac{r}{400}\right)^8$

$\therefore \quad 1+\frac{r}{400} = \left(\frac{4}{3}\right)^{1/8}$

$\Leftrightarrow \quad r = 400\left[\left(\frac{4}{3}\right)^{1/8} - 1\right]$

$= 14.6458$

i.e, need 14.65% p.a.

Q4. P = 4500, over 5 yrs.

Institute A:

$I = 4500 \times 5 \times \frac{12}{100}$

$= 2700$

$\therefore$ Total amount after 5 years is $\$(4500+2700) = \$7200$.

Institute B:

$A = 4500\left(1+\frac{10.6}{100}\right)^5$

$= 7447.1165$

i.e, \$7447.12 after 5 yrs. ← Therefore choose Institute B

Institute C:

10% p.a. compounded monthly

$\Rightarrow \frac{10}{12}\%$ per. months.

Also, 5 yrs = 60 months

$\therefore A = 4500\left(1+\frac{10/12}{100}\right)^{60} = 7403.89$

i.e, \$7403.89 after 5 yrs

Q5. Let $r\%$ be the annual rate of interest. Then, the monthly rate is $\frac{r}{12}\%$.

Two years $\Rightarrow$ 24 months

$$\therefore 9000 = 8000\left(1+\frac{r/12}{100}\right)^{24}$$

$$\Leftrightarrow \quad \frac{9}{8} = \left(1+\frac{r}{1200}\right)^{24}$$

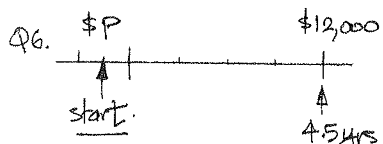
$$\therefore \quad 1+\frac{r}{1200} = \left(\frac{9}{8}\right)^{1/24}$$

$$\Leftrightarrow \quad r = 1200\left[\left(\frac{9}{8}\right)^{\frac{1}{24}}-1\right]$$

$$= 5.90$$

i.e, annual rate $= 5.90\%$.

Q6.



$$8.9\% \text{ p.a} = \frac{8.9}{4}\% \text{ per quarter.}$$

$$= 2.225\%$$

$$4.5 \text{ yrs} = 4.5\times 4$$

$$= 18 \text{ quarters}$$

$$\therefore 12000 = P\left(1+\frac{2.225}{100}\right)^{18}$$

$$\Leftrightarrow \quad P = \frac{12000}{(1.0225)^{18}}$$

$$= 8075.1965$$

i.e, Janice needs to invest \$8075.2

NB: If Janice has to leave the money in account for 5yrs, then she would need to invest $\$\frac{12000}{(1.0225)^{20}}$

i.e, \$7727.50.

However, we have assumed that she can remove her money from the building society without penalties

Q7. 5% p.a compounded semi-annually $\Rightarrow$ 2.5% every six months.

$P = 2000$

1st Jan 2000 $\rightarrow$ 1st July 2002 gives 5 periods.

$$\therefore \text{At 1st July } A = 2000\left(1+\frac{2.5}{100}\right)^5$$

$$= 2262.8164$$

After removing \$1000 there is \$1262.8164 left in account.

This will remain for another 5 periods.

$$\therefore A = 1262.8164\left(1+\frac{2.5}{100}\right)^5$$

$$= 1428.76$$

i.e, \$1428.76 in account.

Q8. $P = 850$

6% p.a. compounded semi-annually

$\Rightarrow$ 3% per period

30 March 1997 $\rightarrow$ 30 March 2004

makes up 14 periods

$$\therefore A = 850\left(1+\frac{3}{100}\right)^{14}$$

$$= 1285.7012.$$

i.e, there was \$1285.70

Q9. $r\%$ p.a. compounded annually

$= \frac{r}{12}\%$ per month.

Six years $\Rightarrow 6\times 12 = 72$ months

$$\therefore 12528.94 = 8000\left(1+\frac{\frac{r}{12}}{100}\right)^{72}$$

$$\Leftrightarrow \left(1+\frac{r}{1200}\right)^{72} = 1.5661175$$

$$\therefore \quad r = 1200\left[1.5661175^{1/72}-1\right]$$

$$= 7.5$$

i.e, 7.5% p.a