How To Graph a Rational Function

1. Find the domain of the function. The domain of a rational function will be all real numbers except when the denominator equals zero. Simply find the zeros of the denominator and those will be the exclusions.

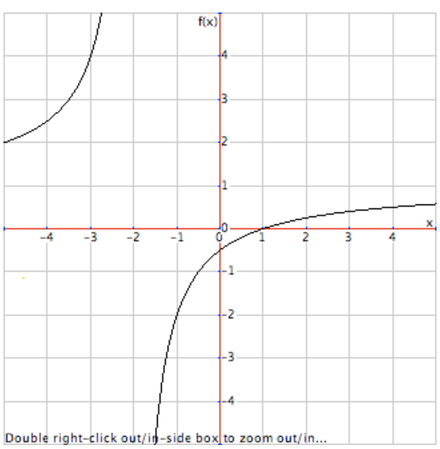
Example: $f(x) = \frac{6}{x-2}$, the domain is $(-\infty, 2) \cup (2, \infty)$.

$f(x) = \frac{5}{2x^2 - x} = \frac{5}{x(2-x)}$, the domain is all reals, $x \neq 0,2$

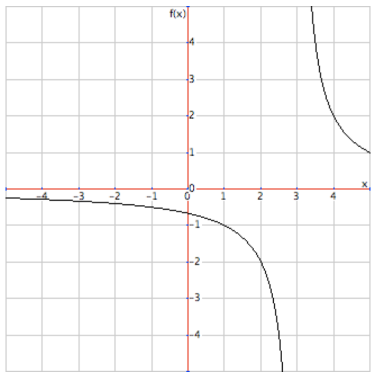
2. Simplify the function if possible by factoring and cancelling out any common factors. If you cancel out any of the factors the value for x- that made that zero will become a hole on the graph and not a V.A.

Example: $f(x) = \frac{x^2 - 1}{(x+1)(x+2)} = \frac{(x+1)(x-1)}{(x+1)(x+2)} = \frac{x-1}{x+2}$

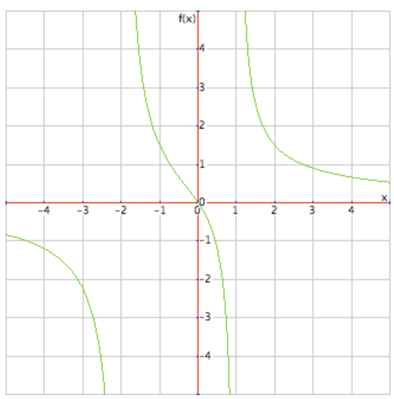
You only need to sketch the graph of the simplified function. If will still be undefined at -1 and -2, but there will only be a V.A at -2. The graph will have a hole at x=-1.



3. Find any Vertical Asymptotes: The remaining factors of the numerator become the vertical asymptotes of the function. Remember as the graph approaches a vertical asymptote from either side, it must approach either positive or negative infinity.



$f(x) = \frac{2}{x-3}$ the function is undefined at x=3 so there is a V.A. Notice how the graph goes towards positive infinity from the right side and negative infinity from the left side.



$f(x) = \frac{3x}{(x+2)(x-1)}$, the function is undefined at x=-2 and x=1, so there will be two vertical asymptotes.

4. Find any horizontal asymptotes: The graph of the function has at most one horizontal asymptote that are found by comparing the degrees of the numerator N(x) and the denominator D(x).

   a. If the the degree of N < the degree of D, the graph of the function has a horizontal asymptote as the line y=0 (the x-axis).

   b. If the degree of N = the degree of D then the function has a horizontal asymptote $y = \frac{a_n}{b_m}$, where a_n is the leading coefficient of the numerator and b_m is the leading coefficient of the denominator.

   c. If the degree of N > than the degree of D the graph has no horizontal asymptote.

5. If there is not a horizontal asymptote, there is a Slant Asymptote. Divide N(x) by D(x), and the Slant Asymptote will the line given by the quotient without the remainder.

$$f(x) = \frac{x^2 - x - 2}{x - 1} \qquad x-1\overline{)x^2 - x - 2} \qquad \begin{array}{r} x - \frac{2}{x-1} \\ x-1\overline{)x^2 - x - 2} \end{array}$$

The line y=x is the Slant Asymptote.

6. Find the x-intercepts and the y-intercepts.
   a. X-intercepts: Set the numerator equal to zero and then find the zeros. If there is no variable in the numerator that will not be any x-intercepts.
   b. Y-intercepts: Make all the x's zero and simplify the fraction. If the y-axis is a V.A., there will not be any y-intercepts, otherwise there must be one.

7. Sketch the curve. To fill in the curve, make sure to plot points on either sides of the V.A. and the x-intercepts.