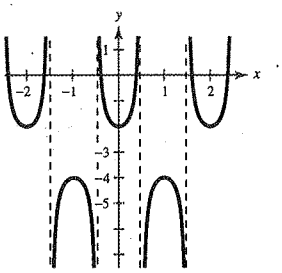
15. $y = \sec \pi x - 3$

Reflect the graph in Exercise #13 about the x-axis and then shift it vertically down three units.

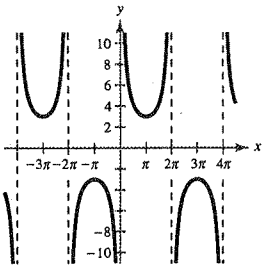


17. $y = 3 \csc \dfrac{x}{2}$

Graph $y = 3 \sin \dfrac{x}{2}$ first.

Period: $\dfrac{2\pi}{1/2} = 4\pi$

One cycle: 0 to 4π



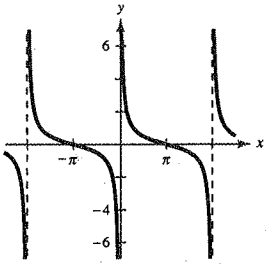
19. $y = \dfrac{1}{2} \cot \dfrac{x}{2}$

Period: $\dfrac{\pi}{1/2} = 2\pi$

Two consecutive asymptotes: $\dfrac{x}{2} = 0 \implies x = 0$

$\dfrac{x}{2} = \pi \implies x = 2\pi$

x	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
y	$\frac{1}{2}$	0	$-\frac{1}{2}$



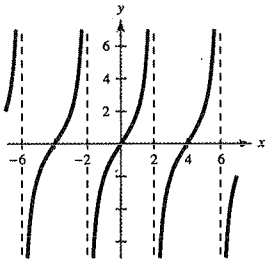
21. $y = 2 \tan \dfrac{\pi x}{4}$

Period: $\dfrac{\pi}{\pi/4} = 4$

Two consecutive asymptotes: $\dfrac{\pi x}{4} = -\dfrac{\pi}{2} \implies x = -2$

$\dfrac{\pi x}{4} = \dfrac{\pi}{2} \implies x = 2$

x	-1	0	1
y	-2	0	2

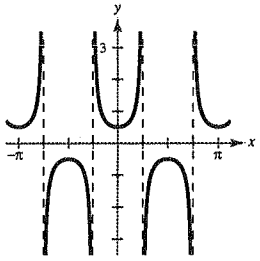


23. $y = \dfrac{1}{2} \sec 2x$

Graph $y = \dfrac{1}{2} \cos 2x$ first.

Period: $\dfrac{2\pi}{2} = \pi$

One cycle: 0 to π



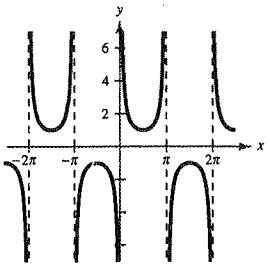
25. $y = \csc(\pi - x)$

Graph $y = \sin(\pi - x)$ first.

Period: 2π

Asymptotes: Set $\pi - x = 0$ and $\pi - x = 2\pi$

$x = \pi$ $\qquad$ $x = -\pi$



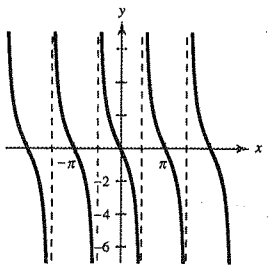
27. $y = 2\cot\left(x - \frac{\pi}{2}\right)$

Period: π

Two consecutive asymptotes: $x - \frac{\pi}{2} = 0 \Rightarrow x = \frac{\pi}{2}$

$$x - \frac{\pi}{2} = \pi \Rightarrow x = \frac{3\pi}{2}$$

x	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$
y	2	0	-2



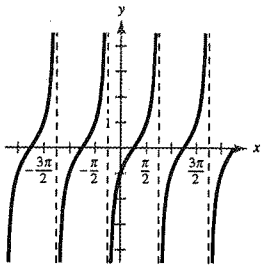
29. $y = \tan\left(x - \frac{\pi}{4}\right)$

Period: π

Two consecutive asymptotes: $x - \frac{\pi}{4} = -\frac{\pi}{2} \Rightarrow x = -\frac{\pi}{4}$

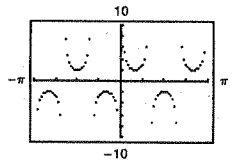
$$x - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow x = \frac{3\pi}{4}$$

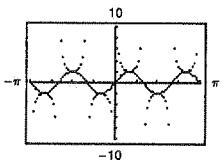
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
y	-1	0	1



31. $y = 2\csc 3x = \frac{2}{\sin(3x)}$

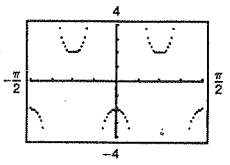
Period: $\frac{2\pi}{3}$

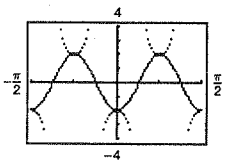




33. $y = -2\sec 4x$

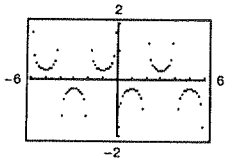
$$= \frac{-2}{\cos 4x}$$

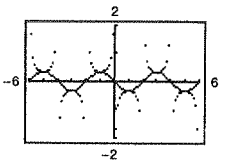




35. $y = \frac{1}{3}\sec\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) = \frac{1}{3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right)}$

Period: 4





37. $\tan x = 1$

$$x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$$

39. $\sec x = -2$

$$x = \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}$$

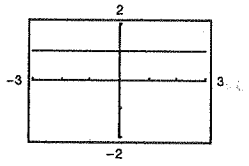
41. The graph of $f(x) = \sec x$ has y-axis symmetry. Thus, the function is even.

43. The function

$$f(x) = \csc 2x = \frac{1}{\sin 2x}$$

has origin symmetry. Thus, the function is odd.

45. $y_1 = \sin x \csc x$ and $y_2 = 1$



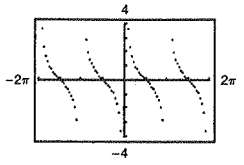
Not equivalent because y_1 is not defined at 0.

$$\sin x \csc x = \sin x\left(\frac{1}{\sin x}\right) = 1, \quad \sin x \neq 0$$

47. $y_1 = \dfrac{\cos x}{\sin x}$ and $y_2 = \cot x = \dfrac{1}{\tan x}$

Equivalent

$$\cot x = \frac{\cos x}{\sin x}$$



49. $f(x) = x \cos x$

As $x \to 0, f(x) \to 0$.

Odd function

$$f\left(\frac{3\pi}{2}\right) = 0$$

Matches graph (d).

51. $g(x) = |x| \sin x$

As $x \to 0, g(x) \to 0$.

Odd function

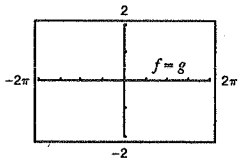
$g(2\pi) = 0$

Matches graph (b).

53. $f(x) = \sin x + \cos\left(x + \dfrac{\pi}{2}\right), g(x) = 0$

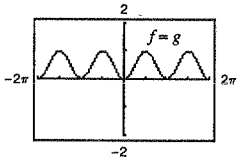
$f(x) = g(x)$

The graph is the line $y = 0$.



55. $f(x) = \sin^2 x, g(x) = \frac{1}{2}(1 - \cos 2x)$

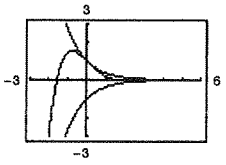
$f(x) = g(x)$



57. $f(x) = e^{-x} \cos x$

Damping factor: e^{-x}

As $x \to \infty, f(x) \to 0$.

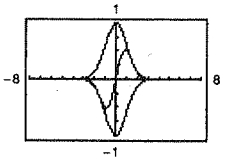


59. $g(x) = e^{-x^2/2} \sin x$

Damping factor: $y = e^{-x^2/2}$

$-e^{-x^2/2} \leq g(x) \leq e^{-x^2/2}$

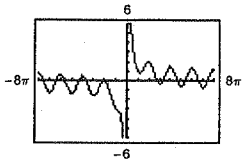
As $x \to \pm\infty, g(x) \to 0$.



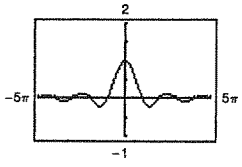
61. $y = \dfrac{6}{x} + \cos x$

As $x \to 0$ from the right, $y \to \infty$.

As $x \to 0$ from the left, $y \to -\infty$.



63.



As $x \to 0, \dfrac{\sin x}{x} \to 1$.