# Chapter 5

# Polynomial and Rational Functions

## Section 5.1

1. $(-2,0)$, $(2,0)$, and $(0,9)$

   x-intercepts: let $y=0$ and solve for $x$

$$9x^2+4(0)=36$$
$$9x^2=36$$
$$x^2=4$$
$$x=\pm 2$$

   y-intercepts: let $x=0$ and solve for $y$

$$9(0)^2+4y=36$$
$$4y=36$$
$$y=9$$

2. True; it has the form $a_n x^n + a_{n-1}x^{n-1}+...+a_1 x + a_0$ where each $a_i$ is a real number and $n$ is a positive integer.

3. down; 4

4. True; for each $x$-intercept we have $y=0$. Therefore, to find the x-intercepts, we solve the equation $y=0$, or $f(x)=0$ since $y=f(x)$.

5. smooth; continuous

6. zero

7. touches

8. True

9. False; the x-intercepts of the graph of a polynomial function are also called zeros of the function.

10. False; the graph of $f$ resembles the graph of $y=3x^4$ for large values of $|x|$.

11. $f(x)=4x+x^3$ is a polynomial function of degree 3.

12. $f(x)=5x^2+4x^4$ is a polynomial function of degree 4.

13. $g(x)=\dfrac{1-x^2}{2}=\dfrac{1}{2}-\dfrac{1}{2}x^2$ is a polynomial function of degree 2.

14. $h(x)=3-\dfrac{1}{2}x$ is a polynomial function of degree 1.

15. $f(x)=1-\dfrac{1}{x}=1-x^{-1}$ is not a polynomial function because it contains a negative exponent.

16. $f(x)=x(x-1)=x^2-x$ is a polynomial function of degree 2.

17. $g(x)=x^{3/2}-x^2+2$ is not a polynomial function because it contains a fractional exponent.

18. $h(x)=\sqrt{x}\left(\sqrt{x}-1\right)=x-x^{1/2}$ is not a polynomial function because it contains fractional exponents.

19. $F(x)=5x^4-\pi x^3+\dfrac{1}{2}$ is a polynomial function of degree 4.

20. $F(x)=\dfrac{x^2-5}{x^3}=x^{-1}-5x^{-3}$ is not a polynomial function because it contains a negative exponent.

21. $$\begin{aligned}G(x)&=2(x-1)^2(x^2+1)=2(x^2-2x+1)(x^2+1)\\&=2(x^4+x^2-2x^3-2x+x^2+1)\\&=2(x^4-2x^3+2x^2-2x+1)\\&=2x^4-4x^3+4x^2-4x+2\end{aligned}$$
    is a polynomial function of degree 4.

22. $$\begin{aligned}G(x)&=-3x^2(x+2)^3=-3x^2(x^3+6x^2+12x+8)\\&=-3x^5-18x^4-36x^3-24x^2\end{aligned}$$
    is a polynomial function of degree 5.



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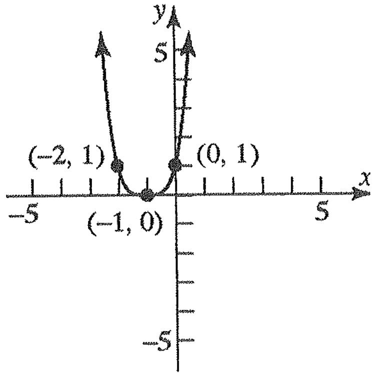
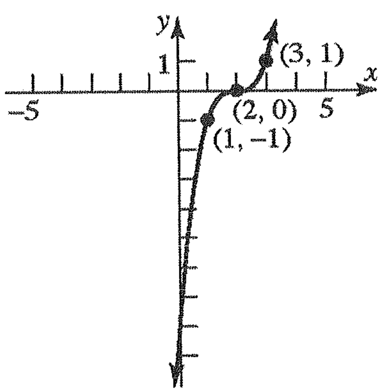
**23.** $f(x)=(x+1)^4$

Using the graph of $y=x^4$, shift the graph horizontally, 1 unit to the left.



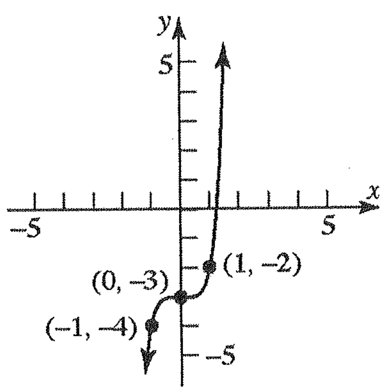

**24.** $f(x)=(x-2)^5$

Using the graph of $y=x^5$, shift the graph horizontally to the right 2 units.



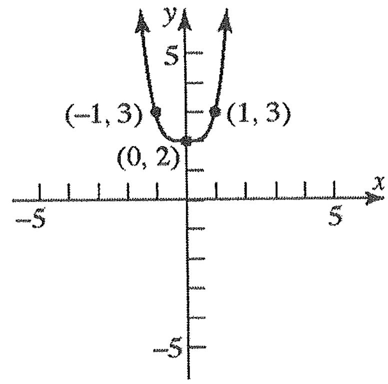

**25.** $f(x)=x^5-3$

Using the graph of $y=x^5$, shift the graph vertically, 3 units down.



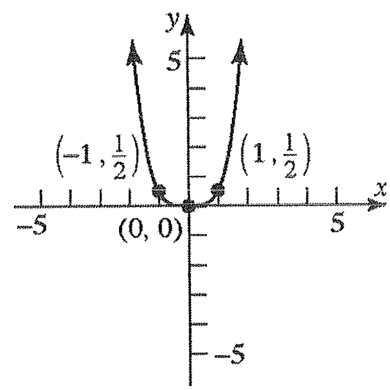

**26.** $f(x)=x^4+2$

Using the graph of $y=x^4$, shift the graph vertically up 2 units.



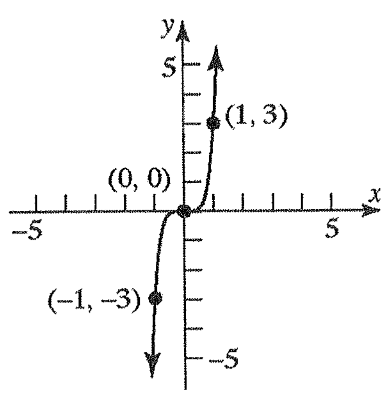

**27.** $f(x)=\frac{1}{2}x^4$

Using the graph of $y=x^4$, compress the graph vertically by a factor of $\frac{1}{2}$.




**28.** $f(x)=3x^5$

Using the graph of $y=x^5$, stretch the graph vertically by a factor of 3.



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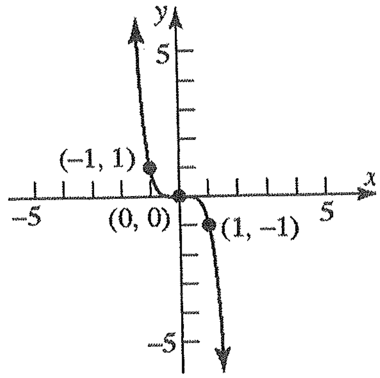
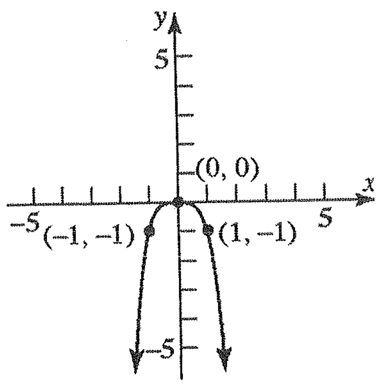
**29.** $f(x) = -x^5$

Using the graph of $y = x^5$, reflect the graph about the $x$-axis.



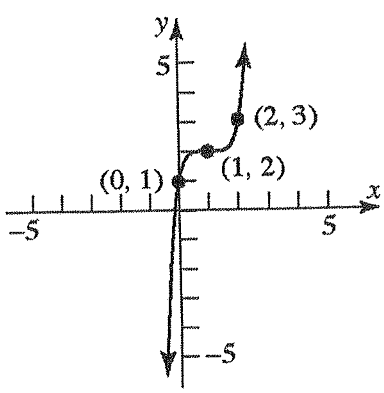

**30.** $f(x) = -x^4$

Using the graph of $y = x^4$, reflect the graph about the $x$-axis.



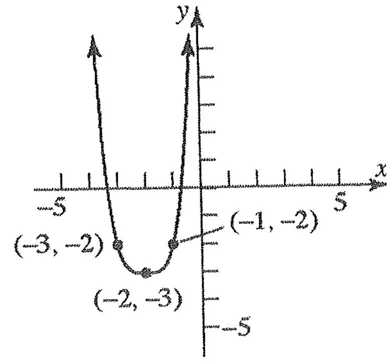

**31.** $f(x) = (x-1)^5 + 2$

Using the graph of $y = x^5$, shift the graph horizontally, 1 unit to the right, and shift vertically 2 units up.



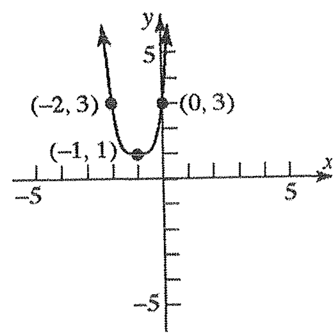

**32.** $f(x) = (x+2)^4 - 3$

Using the graph of $y = x^4$, shift the graph horizontally left 2 units, and shift vertically down 3 units.



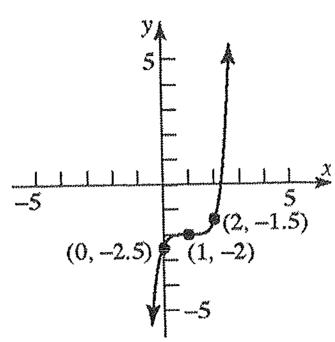

**33.** $f(x) = 2(x+1)^4 + 1$

Using the graph of $y = x^4$, shift the graph horizontally, 1 unit to the left, stretch vertically by a factor of 2, and shift vertically 1 unit up.




**34.** $f(x) = \frac{1}{2}(x-1)^5 - 2$

Using the graph of $y = x^5$, shift the graph horizontally 1 unit to the right, compress vertically by a factor of $\frac{1}{2}$, and shift vertically down 2 units.



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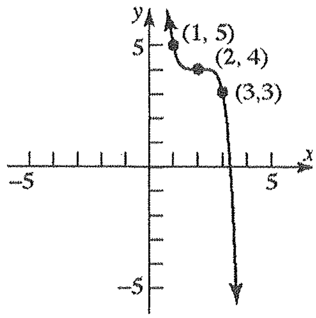
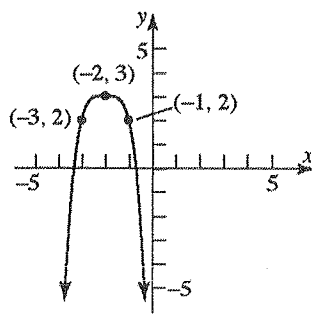
**35.** $f(x) = 4 - (x-2)^5 = -(x-2)^5 + 4$

Using the graph of $y = x^5$, shift the graph horizontally, 2 units to the right, reflect about the $x$-axis, and shift vertically 4 units up.




**36.** $f(x) = 3 - (x+2)^4 = -(x+2)^4 + 3$

Using the graph of $y = x^4$, shift the graph horizontally, 2 units to the left, reflect about the $x$-axis, and shift vertically 3 units up.




**37.** $f(x) = a\big(x-(-1)\big)(x-1)(x-3)$

For $a = 1$:

$$f(x) = (x+1)(x-1)(x-3) = \left(x^2 - 1\right)(x-3)$$
$$= x^3 - 3x^2 - x + 3$$

**38.** $f(x) = a\big(x-(-2)\big)(x-2)(x-3)$

For $a = 1$:

$$f(x) = (x+2)(x-2)(x-3) = \left(x^2 - 4\right)(x-3)$$
$$= x^3 - 3x^2 - 4x + 12$$

**39.** $f(x) = a\big(x-(-3)\big)(x-0)(x-4)$

For $a = 1$:

$$f(x) = (x+3)(x)(x-4) = \left(x^2 + 3x\right)(x-4)$$
$$= x^3 - 4x^2 + 3x^2 - 12x$$
$$= x^3 - x^2 - 12x$$

**40.** $f(x) = a\big(x-(-4)\big)(x-0)(x-2)$

For $a = 1$:

$$f(x) = (x+4)(x)(x-2) = \left(x^2 + 4x\right)(x-2)$$
$$= x^3 - 2x^2 + 4x^2 - 8x$$
$$= x^3 + 2x^2 - 8x$$

**41.** $f(x) = a\big(x-(-4)\big)\big(x-(-1)\big)(x-2)(x-3)$

For $a = 1$:

$$f(x) = (x+4)(x+1)(x-2)(x-3)$$
$$= \left(x^2 + 5x + 4\right)\left(x^2 - 5x + 6\right)$$
$$= x^4 - 5x^3 + 6x^2 + 5x^3 - 25x^2 + 30x + 4x^2 - 20x + 24$$
$$= x^4 - 15x^2 + 10x + 24$$

**42.** $f(x) = a\big(x-(-3)\big)\big(x-(-1)\big)(x-2)(x-5)$

For $a = 1$:

$$f(x) = (x+3)(x+1)(x-2)(x-5)$$
$$= \left(x^2 + 4x + 3\right)\left(x^2 - 7x + 10\right)$$
$$= x^4 - 7x^3 + 10x^2 + 4x^3 - 28x^2$$
$$+ 40x + 3x^2 - 21x + 30$$
$$= x^4 - 3x^3 - 15x^2 + 19x + 30$$

**43.** $f(x) = a\big(x-(-1)\big)(x-3)^2$

For $a = 1$:

$$f(x) = (x+1)(x-3)^2$$
$$= (x+1)\left(x^2 - 6x + 9\right)$$
$$= x^3 - 6x^2 + 9x + x^2 - 6x + 9$$
$$= x^3 - 5x^2 + 3x + 9$$

**44.** $f(x) = a\big(x-(-2)\big)^2(x-4)$

For $a = 1$:

$$f(x) = (x+2)^2(x-4)$$
$$= \left(x^2 + 4x + 4\right)(x-4)$$
$$= x^3 - 4x^2 + 4x^2 - 16x + 4x - 16$$
$$= x^3 - 12x - 16$$

**45.** **a.** The real zeros of $f(x) = 3(x-7)(x+3)^2$ are: 7, with multiplicity one; and –3, with multiplicity two.

**b.** The graph crosses the x-axis at 7 (odd multiplicity) and touches it at –3 (even multiplicity).



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c. Near $-3$: $f(x) \approx -30(x+3)^2$;
Near 7: $f(x) \approx 300(x-7)$

d. $n-1=3-1=2$

e. The function resembles $y=3x^3$ for large values of $|x|$.

**46. a.** The real zeros of $f(x)=4(x+4)(x+3)^3$ are: $-4$, with multiplicity one; and $-3$, with multiplicity three.

b. The graph crosses the $x$-axis at $-4$ and at $-3$ (odd multiplicities).

c. Near $-4$: $f(x) \approx -4(x+4)$;
Near $-3$: $f(x) \approx 4(x+3)^3$

d. $n-1=4-1=3$

e. The function resembles $y=4x^4$ for large values of $|x|$.

**47. a.** The real zeros of $f(x)=4\left(x^2+1\right)(x-2)^3$ is: 2, with multiplicity three.
$x^2+1=0$ has no real solution.

b. The graph crosses the $x$-axis at 2 (odd multiplicity).

c. Near 2: $f(x) \approx 20(x-2)^3$

d. $n-1=5-1=4$

e. The function resembles $y=4x^5$ for large values of $|x|$.

**48. a.** The real zeros of $f(x)=2\left(x-3\right)(x+4)^3$ are: 3, with multiplicity one; and $-4$, with multiplicity three.

b. The graph crosses the x-axis at 3 and at $-4$ (odd multiplicities).

c. Near $-4$: $f(x) \approx -14(x+4)^3$;
Near 3: $f(x) \approx 686(x-3)$

d. $n-1=4-1=3$

e. The function resembles $y=2x^4$ for large values of $|x|$.

**49. a.** The real zero of
$f(x)=-2\left(x+\frac{1}{2}\right)^2\left(x^2+4\right)^2$ is: $-\frac{1}{2}$, with multiplicity two. $x^2+4=0$ has no real solution.

b. The graph touches the x-axis at $-\frac{1}{2}$ (even multiplicity).

c. Near $-\frac{1}{2}$: $f(x) \approx -36.125\left(x+\frac{1}{2}\right)^2$

d. $n-1=6-1=5$

e. The function resembles $y=-2x^6$ for large values of $|x|$.

**50. a.** The real zeros of $f(x)=\left(x-\frac{1}{3}\right)^2\left(x-1\right)^3$ are: $\frac{1}{3}$, with multiplicity two; and 1, with multiplicity 3.

b. The graph touches the $x$-axis at $\frac{1}{3}$ (even multiplicity), and crosses the x-axis at 1 (odd multiplicity).

c. Near $\frac{1}{3}$: $f(x) \approx -\frac{8}{27}\left(x-\frac{1}{3}\right)^2$;
Near 1: $f(x) \approx \frac{4}{9}\left(x-1\right)^3$

d. $n-1=5-1=4$

e. The function resembles $y=x^5$ for large values of $|x|$.

**51. a.** The real zeros of $f(x)=(x-5)^3(x+4)^2$ are: 5, with multiplicity three; and $-4$, with multiplicity two.

b. The graph crosses the x-axis at 5 (odd multiplicity) and touches it at $-4$ (even multiplicity).

c. Near $-4$: $f(x) \approx -729\left(x+4\right)^2$;
Near 5: $f(x) \approx 81\left(x-5\right)^3$

d. $n-1=5-1=4$



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e. The function resembles $y = x^5$ for large values of $|x|$.

52. a. The real zeros of $f(x) = \left(x+\sqrt{3}\right)^2 (x-2)^4$ are: $-\sqrt{3}$, with multiplicity two; and 2, with multiplicity four.

b. The graph touches the x-axis at $-\sqrt{3}$ and at 2 (even multiplicities).

c. Near $-\sqrt{3}$: $f(x) \approx 194\left(x+\sqrt{3}\right)^2$;

Near 2: $f(x) \approx 13.93(x-2)^4$

d. $n-1 = 6-1 = 5$

e. The function resembles $y = x^6$ for large values of $|x|$.

53. a. $f(x) = 3\left(x^2+8\right)\left(x^2+9\right)^2$ has no real zeros. $x^2+8=0$ and $x^2+9=0$ have no real solutions.

b. The graph neither touches nor crosses the x-axis.

c. No real zeros

d. $n-1 = 6-1 = 5$

e. The function resembles $y = 3x^6$ for large values of $|x|$.

54. a. $f(x) = -2\left(x^2+3\right)^3$ has no real zeros. $x^2+3=0$ has no real solutions.

b. The graph neither touches nor crosses the x-axis.

c. No real zeros

d. $n-1 = 6-1 = 5$

e. The function resembles $y = -2x^6$ for large values of $|x|$.

55. a. The real zeros of $f(x) = -2x^2(x^2-2)$ are: $-\sqrt{2}$ and $\sqrt{2}$ with multiplicity one; and 0, with multiplicity two.

b. The graph touches the x-axis at 0 (even multiplicity) and crosses the x-axis at $-\sqrt{2}$ and $\sqrt{2}$ (odd multiplicities).

c. Near $-\sqrt{2}$: $f(x) \approx 11.31\left(x+\sqrt{2}\right)$;

Near 0: $f(x) \approx 4x^2$;

Near $\sqrt{2}$: $f(x) \approx -11.31\left(x-\sqrt{2}\right)$

d. $n-1 = 4-1 = 3$

e. The function resembles $y = -2x^4$ for large values of $|x|$.

56. a. The real zeros of $f(x) = 4x(x^2-3)$ are: $-\sqrt{3}$, $\sqrt{3}$ and 0, with multiplicity one.

b. The graph crosses the x-axis at $-\sqrt{3}$, $\sqrt{3}$ and 0 (odd multiplicities).

c. Near $-\sqrt{3}$: $f(x) \approx 24\left(x+\sqrt{3}\right)$;

Near 0: $f(x) \approx -12x$;

Near $\sqrt{3}$: $f(x) \approx 24\left(x-\sqrt{3}\right)$

d. $n-1 = 3-1 = 2$

e. The function resembles $y = 4x^3$ for large values of $|x|$.

57. Could be; zeros: $-1, 1, 2$; minimum degree = 3

58. Could be; zeros: $-1, 2$; minimum degree = 4

59. Can't be; not continuous at $x = -1$

60. Can't be; not smooth at $x = 0$

61. c, e, f

62. c, e, f

63. c, e

64. d, f

65. $f(x) = (x-1)^2$

a. $y$-intercept: $f(0) = (0-1)^2 = 1$

$x$-intercept: solve $f(x) = 0$

$(x-1)^2 = 0 \Rightarrow x = 1$

b. The graph touches the $x$-axis at $x = 1$, since this zero has multiplicity 2.

c. Degree is 2; The function resembles $y = x^2$ for large values of $|x|$.

d. 1



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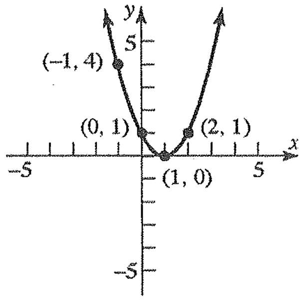
e. Near 1: $f(x) \approx (x-1)^2$

f. Graphing:




66. $f(x) = (x-2)^3$

a. $y$-intercept: $f(0) = (0-2)^3 = -8$
$x$-intercept: solve $f(x) = 0$
$0 = (x-2)^3$
$x = 2$

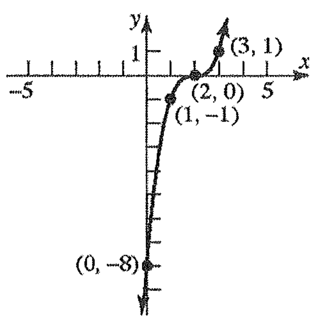
b. crosses $x$-axis at $x = 2$

c. Degree is 3. The function resembles $y = x^3$ for large values of $|x|$.

d. 2

e. Near 2: $f(x) \approx (x-2)^3$

f. Graphing by hand




67. $f(x) = x^2(x-3)$

a. $y$-intercept: $f(0) = 0^2(0-3) = 0$
$x$-intercepts: solve $f(x) = 0$
$0 = x^2(x-3)$
$x = 0,\ x = 3$

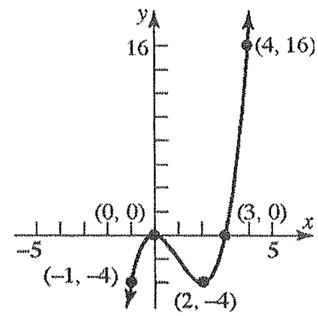
b. touches $x$-axis at $x = 0$; crosses $x$-axis at $x = 3$

c. Degree is 3. The function resembles $y = x^3$ for large values of $|x|$.

d. 2

e. Near 0: $f(x) \approx -3x^2$; Near 3: $f(x) \approx 9(x-3)$

f. Graphing by hand:




68. $f(x) = x(x+2)^2$

a. $y$-intercept: $f(0) = 0(0+2)^2 = 0$
$x$-intercepts: solve $f(x) = 0$
$0 = x(x+2)^2$
$x = 0, -2$

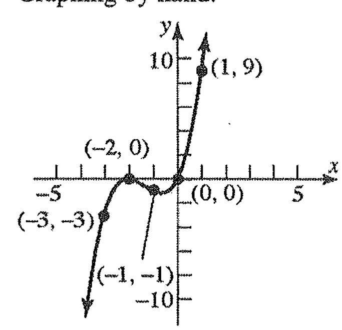
b. touches $x$-axis at $x = -2$; crosses $x$-axis at $x = 0$

c. Degree is 3. The function resembles $y = x^3$ for large values of $|x|$.

d. 2

e. Near -2: $f(x) \approx -2(x+2)^2$; Near 0: $f(x) \approx 4x$

f. Graphing by hand:



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